Effect of Mean Stress in High-temperature Fatigue

Energy-based approach is considered for crack growth and the result is used to explain the experimental investigations carried out to study the effect of mean stress on the fatigue behavior of alpha brass at elevated temperature

by V.M. Radhakrishnan

ABSTRACT—Investigations have been carried out to study the effect of mean stress and temperature on the fatigue behavior of alpha brass. Introduction of mean stress reduces the fatigue life of the material, the reduction being very much pronounced in the temperature region of 0.5 T_m. Theoretical analysis based on hysteresis energy yields relations which describe well the experimental findings.

Notations

a, A_1, A_2 = constants
B = constant
$C_1, C_2 \ldots$ = constants
K_1 = maximum stress-intensity factor
K_2 = minimum stress-intensity factor
K_a = stress-intensity factor corresponding to σ_a
l = half crack length
l_o = fictitious crack length corresponding to σ_u
l_c = critical crack length
N = number of cycles
N_f = number of cycles to fracture
p, q = constants
Q = activation energy
R = stress ratio
T = temperature in absolute scale
T_m = melting temperature
δw = hysteresis energy of bulk material
δw^+ = hysteresis energy at crack tip
β = temperature-dependent constant
σ_1 = maximum applied stress
σ_a = alternating component of the stress
σ_m = mean stress
σ_p^+ = stress at crack tip
σ_u = ultimate strength
ϵ_p = plastic strain
ϵ_p^+ = plastic strain at crack tip

Introduction

The effect of mean stress on the crack propagation and fatigue failure has been analyzed by many research workers.[1-6] Frost and Greenan[2,3] have shown that for both zero mean stress and the general-tensile stress-loading cycle $\sigma_m \pm \sigma_a$, the parameter $\sigma_a^3\, l$ decides the growth of the crack, and if the parameter is less than C_o, a material constant, the crack will remain dormant. Broek and Schijve[4] have suggested that the influence of mean stress on the crack growth must be important and that the general relation can be given in the form

$$dl/dN = C_1\, \sigma_m^a\, \sigma_a^b \tag{1}$$

where the exponents a and b are constants. b depends on the crack length and also on mean stress. They have developed a general equation applicable to 2024 T3 and 7075 T6 alloys in the form

$$dl/dN = C_2 \exp(-C_3 R)\, K_1^3 \tag{2}$$

As the stress ratio R increases, this relation means that the growth rate decreases exponentially. Roberts and Erdogan[6] have proposed a crack-propagation model that accounts for the effect of mean stress. The model yields the relation

$$dl/dN = C_4\, (1 + D)^{2a_1}\, (K_a)^{2(a_1+a_2)} \tag{3}$$

where $D = (K_1 + K_2)/K_a$ and a_1 and a_2 are experimentally evaluated exponents.

Walker[7,8] has considered the general equation of Broek and Schijve in the following form:

$$dl/dN = \text{func}\, (K_1)^c\, (K_a)^b\, B \tag{4}$$

and suggested that it can be written as

$$dl/dN = \text{func}\, [(\sigma_1)^{1-p}\, (\sigma_a)^p\, \sqrt{2lB}\,]^q \tag{5}$$

where $1 - p = c/c + b$, $q = c + b$. The value of p is around 2. The crack-growth data for various materials have been analyzed by Frost *et al.*,[9] who prefer the parameter $\sigma_a^3 l$ to K_a as a satisfactory means to explain the crack growth.

In the following, an energy-based approach is considered for crack growth and the result is used to explain the experimental investigations carried out to study the effect of mean stress on the fatigue behavior of alpha brass at elevated temperature.

Theoretical Considerations

Many theories for the propagation of fatigue cracks have been developed based on the linear fracture-

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mechanics approach.[10,11] However, when the stress level is comparatively high ($\sigma_a > \sigma_{ys}$) or when the temperature is in the elevated region, the fracture-mechanics approach may not be basically valid. In such a case, the problem may probably be analyzed from hysteresis strain-energy considerations. The fatigue life can be divided into two parts: (a) crack nucleation and (b) crack propagation up to the critical stage. The nucleation period is dependent on the stress level and temperature. At elevated temperature, its contribution to total life may be negligible. After a crack has formed, the stress and strain near the crack tip are intensified due to the notch effect. The theoretical stress-concentration factor K_t for a sharp crack, which is very small compared to the width of the specimen can be written as

$$K_t = 1 + 2\sqrt{l/\varrho} \approx 2\sqrt{l/\varrho} \text{ for } l >> \varrho \qquad (6)$$

where l is the semi-crack length and ϱ the tip radius. Due to plastic deformation, the severity of the stress at the notch is not as great as is given by the eq (6), but it is less. If σ_p^*, ϵ_p^* are the stress and strain near the crack, then the stress and strain concentration factors, K_p and K_e, are related by[12]

$$K_t^2 = K_p K_e = \frac{\sigma_p^*}{\sigma} \frac{\epsilon_p^*}{\epsilon_p} \qquad (7)$$

where σ, ϵ_p are the applied stress and strain away from the crack.

Consider a miniature tensile specimen, as shown in Fig. 1, just ahead of the crack, which also undergoes the stress-strain cycle of $\sigma_p^* \epsilon_p^*$, when the bulk material is subjected to σ_a and ϵ_p. Damage accumulation in the form of microvoids takes place in the miniature tensile specimen and this damage per unit volume of the material at the crack tip, will be proportional to the plastic-energy density δw^* near the crack tip. After the completion of δN number of cycles (which may be equal to one, as one-to-one correspondence has been established between the fatigue striations and the number of cycles), the miniature specimen fails and the crack propagates through the width of the specimen δl. The width of the specimen over which damage accumulation takes place, will depend very much on the alternating component σ_a, which causes fatigue damage, and the mean component σ_m which causes the creep damage. Also the type of fatigue, low cycle or high cycle, will play a part, the low cycle being more damaging than the high cycle. The failure mechanism is mainly due to the movement of dislocations and vacancies, which are very much activated by increase in temperature. Hence, the crack propagation at relatively high temperature (which is below the recrystallization temperature where the structure remains more or less stable), can be written as

$$\delta l/\delta N = A_1 \delta w^* (\sigma_a + \beta\sigma_m)^\alpha \exp(-Q/kT) \qquad (8)$$

where β is a constant, which decides the contribution of the mean stress to the damage accumulation, and Q is the activation energy for vacancy movement in the miniature specimen, required for the void formation. The exponent α will depend on the type of fatigue cycling—its value being more predominant at low-cycle fatigue. The quantities β and α will determine the interaction of creep and low-cycle fatigue.

The hysteresis energy absorbed either by the miniature

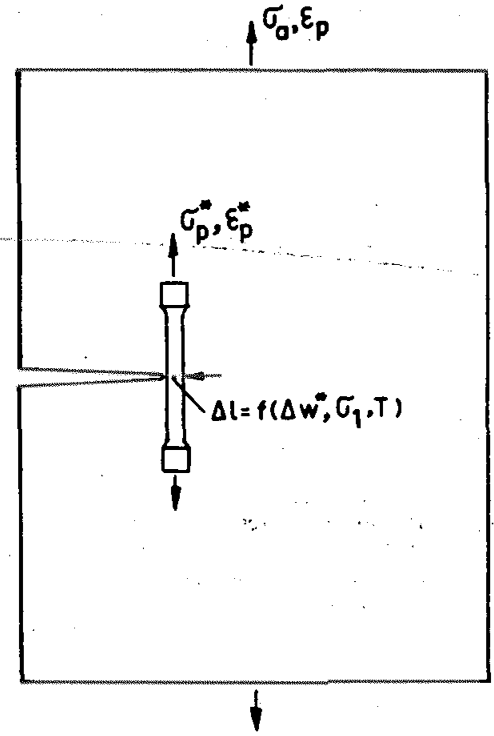

Fig. 1—Crack-propagation model

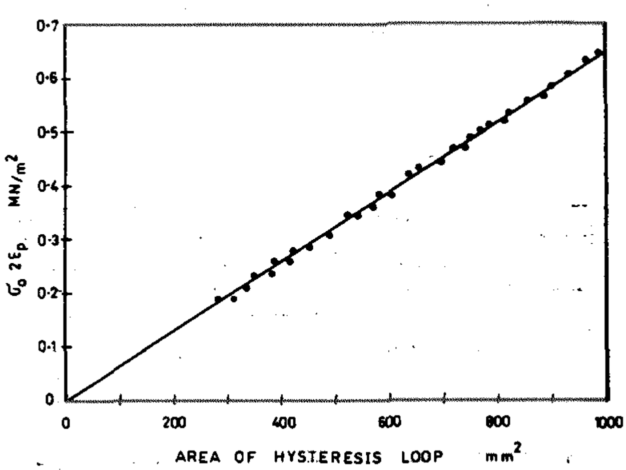

Fig. 2—Relation between area of hysteresis loop and the product $\sigma_a \epsilon_p$

specimen or by the bulk material is a function of the alternating stress and the corresponding plastic strain. A typical relation between the hysteresis energy absorbed and the product $\sigma_a \epsilon_p$ for annealed copper is shown in Fig. 2.[13] Hence, the hysteresis energy absorbed can be written as

$$\delta w = A_2 \sigma_a \epsilon_p \quad \text{and}$$
$$\delta w^* = A_2 \sigma_a^* \epsilon_p^* \qquad (9)$$

From eq (7) we have

$$K_t^2 = K_p K_e = \frac{\sigma_p^* \epsilon_p^*}{\sigma_a \epsilon_p} = \frac{\delta w^*}{\delta w} = (2\sqrt{l/\varrho})^2 \qquad (10)$$

Substituting the value δw^{*} in eq (8) by the term $\delta w^{*} = \delta w\,(2\sqrt{l/\varrho})^2$, obtained from the above relation the crack growth rate can be written as

$$\delta l/\delta N = dl/dN = A_3\,\delta w\,l\,(\sigma_a + \beta\,\sigma_m)^{\alpha} \exp(-Q/kT) \quad (11)$$

Integrating the above relation from l_o, a very small crack at the time of nucleation, to l and the corresponding number of cycles N, we get

$$\log(l/l_o) = A_3\,\delta w\,(\sigma_a + \beta\,\sigma_m)^{\alpha} \exp(-Q/kT).\,N. \quad (12)$$

When the crack length l reaches the critical stage l_c, the total cycles will be N_f. At comparatively high temperature, the nucleation period will be very much reduced and, hence, it can be neglected.

Fatigue fracture is the result of the propagation of a single dominant crack up to the critical level l_c and the failure is catastrophic; i.e., as soon as the crack reaches l_c, its propagation will be very rapid, just as brittle fracture and, hence, a Griffith type of relation can be assumed to govern the final crack length in the form

$$\sigma_1\sqrt{l_c} = \text{constant} = \sigma_u\sqrt{l_o} \quad (13)$$

where l_o is a fictitious crack length assumed to correspond to the ultimate strength σ_u and here it is identified with the length of the fatigue crack at the time of nucleation. So, with eqs (12) and (13), we get the total number of cycles to fracture N_f as

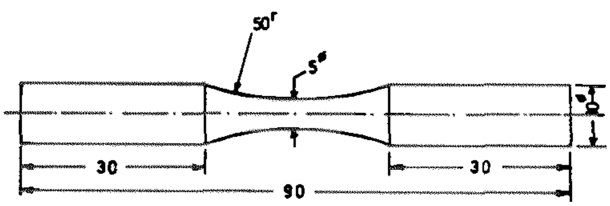

Fig. 3—Dimensions of the specimen

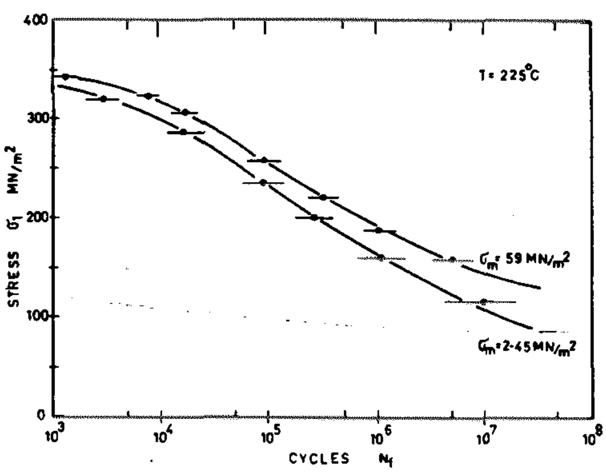

Fig. 4—S-N relation at 225°C

$$N_f = \frac{A_4 \log(\sigma_u/\sigma_1)\exp(Q/kT)}{\sigma_a\,\epsilon_p\,(\sigma_a + \beta\,\sigma_m)^{\alpha}} \quad (14)$$

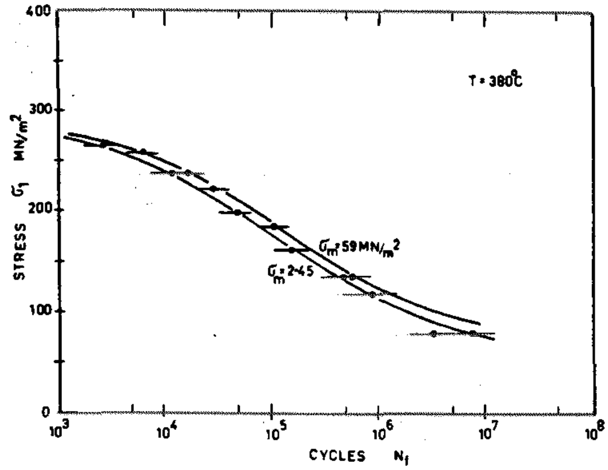

Fig. 5—S-N relation at 380°C

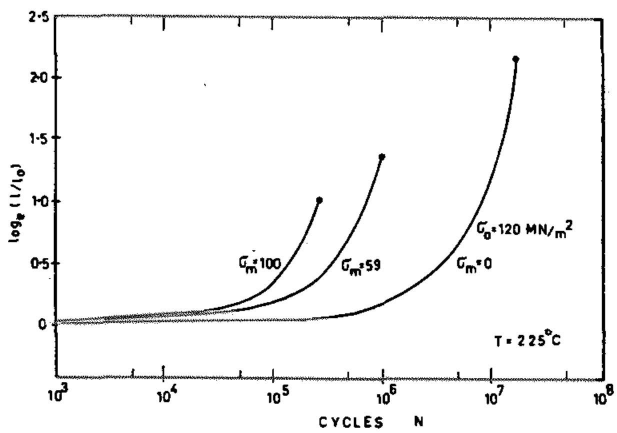

Fig. 6—Crack growth at 225°C

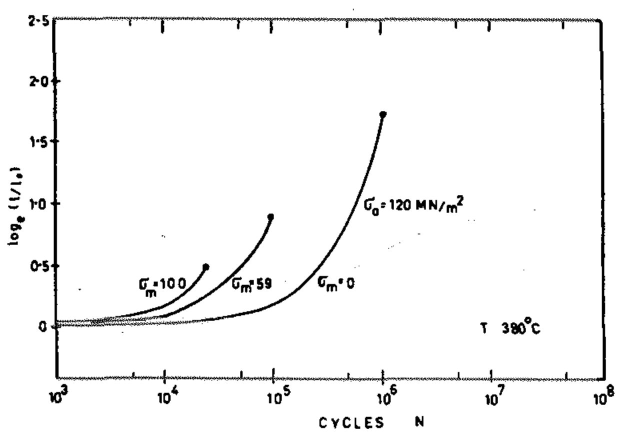

Fig. 7—Crack growth at 380°C

Experimental

The material investigated was alpha brass (Cu : 70.4 %, Zn : 29.6 %, and Pb : nil), annealed at 400°C for 6 h. The ultimate strength at room temperature was 380 MN/m². A rotary bending-fatigue-testing machine with a provision for the application of mean stress was used for the investigation. The frequency of stress cycling was 1500/min. The dimensions of the specimen are given in Fig. 3.

Fatigue experiments were carried out at specimen temperatures of 225°C, 275°C, 325°C and 380°C. The temperature could be controlled within ±2°C at the lower ranges and within ±3°C at higher ranges. The minimum mean stress σ_m that could be applied was 2.45 MN/m². The mean stress was varied in stages of 2.45 MN/m², 19.6 MN/m², 39.2 MN/m² and 59 MN/m². The S-N curves were established at the four temperatures with the previously mentioned mean stresses. At each stress level, at least four specimens were tested.

Results and Discussion

Figures (4) and (5) show the S-N relation at the two extreme temperatures, namely, 225°C and 380°C. The relation between the alternating stress σ_a and the corresponding plastic strain ϵ_p can be given by $\sigma_a = \sigma_o \epsilon_p^{n'}$ where n' is the cyclic strain-hardening exponent. The gross plastic deformation in high-cycle fatigue will normally be small and hence a parabolic strain-hardening law for small values of ϵ_p can be assumed, i.e., $n' = 0.5$.

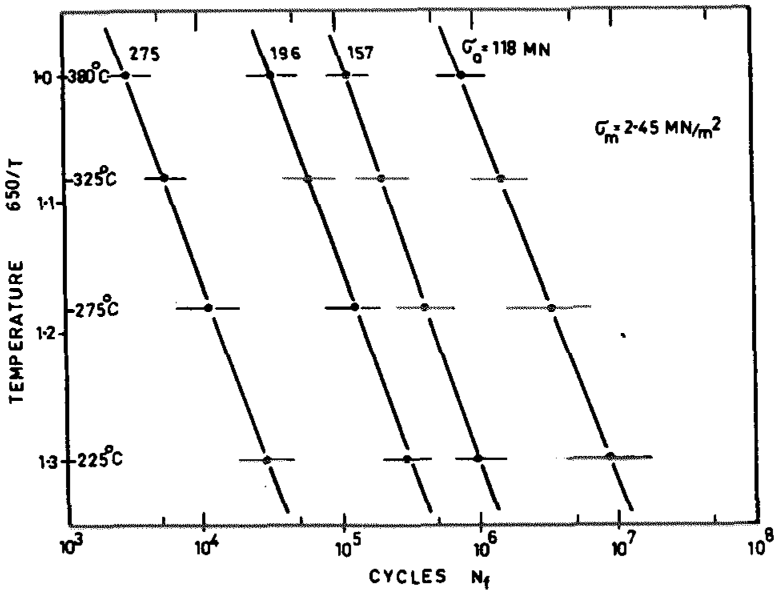

Fig. 8—Relation between $1/T$ and N_f for mean stress σ_m = 2.45 MN/m²

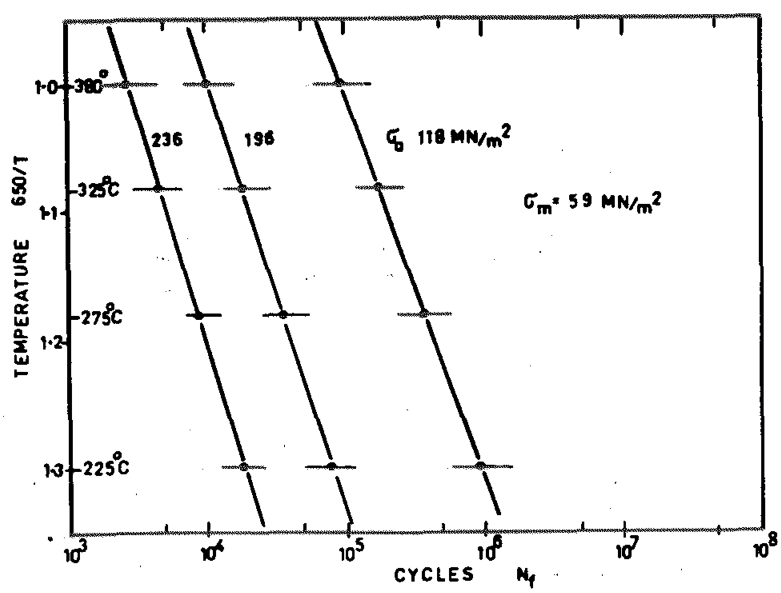

Fig. 9—Relation between $1/T$ and N_f for mean stress σ_m = 59.0 MN/m²

Also, in high-cycle fatigue, where the damage will not be so severe as in low-cycle fatigue, the exponent α can be taken as unity, so that the analysis is only for high-cycle fatigue with mean stress. With these assumptions, the relation (14) can be written as

$$N_f = \frac{A \log (\sigma_u / \sigma_1) \exp (Q/kT)}{\sigma_a^3 (\sigma_a + \beta \sigma_m)} \qquad (15)$$

For a given temperature, the term $A \exp (Q/kT)$ is a constant. The values of the ultimate strength are 353 MN/m^2 and 280 MN/m^2 at 225°C and 380°C, respectively. The constant β, as discussed earlier, determines the contribution of the mean stress to damage accumulation, and its value will increase with increase in temperature. Assigning an arbitrary value of $\beta = 2$ at 225°C and $\beta = 5$ at 380°C, the quantity $A \exp (Q/kT)$ has been evaluated by choosing a test point. The values of this term for these two temperatures were calculated as 19×10^{14} and 5×10^{14}, respectively. The full lines in Figs. (4) and (5) show the calculated values according to the above relation, and the theoretical prediction appears to fit well with the experimental data. The contribution of the mean stress to the damage accumulation could be taken into account only empirically through the constant β. As such, eq (15) contains two arbitrary constants which are to be determined by two test points. The probable advantage in the above relation is that with only two experimentally determined constants, the fatigue life could be estimated over a range of mean stresses.

Figures (6) and (7) show the growth of the crack with the number of fatigue cycles, according to relation (12). The critical crack length is assumed to be governed by a Griffith-type relation, given in the form $\sigma_1 \sqrt{l_c}$ = constant. It can be seen from the graphs that the growth is accelerated with the increase in mean stress at a given alternating stress value. These results are similar to what has been suggested by Broek and Schijve.[4] It can also be noted that the growth rate of the crack increases with increase in temperature for given stress conditions. The growth rates have been predicted for different mean stresses with one value of the test point to evaluate the term $A \exp (Q/kT)$.

Figures (8) and (9) show the relation between the number of cycles to failure and the temperature ($1/T$) on a semi-log plot. Though the constant β is temperature dependent, as it is brought into the log term, its contribution will be negligible and, hence, the plot of $1/T$ vs. $\log N_f$ yields a straight-line relation according to

$$\log N_f = Q/kT + A_4 \qquad (16)$$

The activation energy Q depends on the process which governs the damage-accumulation mechanism and, hence, may be a function of stress and temperature. Due to inherent nature of scatter in data in fatigue, the exact dependence of the activation energy on stress and temperature could not be evaluated. The average value of the activation energy is determined to be around 10 Kcal/mole.

Figure (10) shows the relation between the alternating component σ_a and the mean component σ_m of the applied stress, which gives a life period of 10^6 cycles.

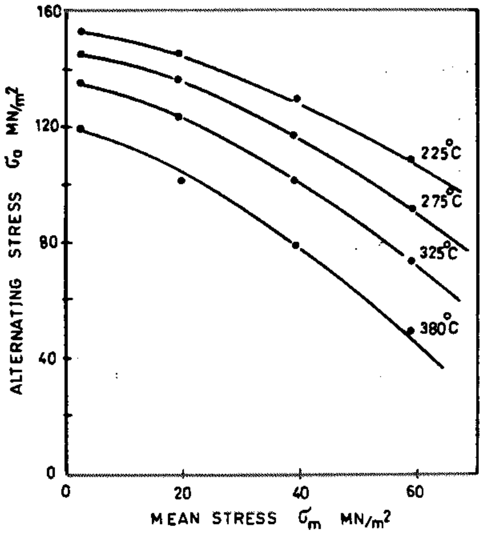

Fig. 10—Relation between alternating stress and mean stress for a life of 10^6 cycles

Conclusions

From the experimental investigations carried out on alpha brass, it has been observed that the introduction of mean stress reduces the fatigue life of the material, the reduction being very much pronounced in the temperature region of 0.5 T_m. Theoretical analysis based on hysteresis energy yields relations which appear to describe well the experimental findings. The theory also shows increased crack-propagation rate with increase in mean stress and temperature. The fracture life N_f bears a straight-line relation with $1/T$ on a semi-log plot and the activation energy for the process of damage accumulation is around 10 Kcal/mole. With increasing temperature, the alternating stress required to give a finite life reduces and this reduction is much more pronounced at higher values of the mean stress.

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