Facts and Fallacies of STRESS DETERMINATION

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Facts and Fallacies of

EVERY engineer is taught how to solve for stress in loaded members and, after becoming reasonably proficient at stress problems, he is advised that it is a pretty good idea to assume that his answers are from 50 to 90% wrong. Of course, he is not advised in these words or with this emphasis. More elegantly, a 50% probable error becomes a factor of safety of 2, a 90% probable error becomes a factor of safety of 10; and thus, the greater the doubt, the safer we feel. The fact is that no means are yet available for reliable determination of stresses in dynamically loaded, highly stressed machine parts. Stress calculations are entirely inadequate except for a few special cases for which empirical formulas have been constructed from extensive service and experimental data. Photo-elastic analyses can be made to reveal major stress patterns but do not provide useful quantitative values of stress. Extensometer readings, brittle lacquers, and so on, likewise can show major stress patterns but fail to provide reliable quantitative stress values. Even if true stress could be found, the permissible working stress cannot be judged from fatigue tests on laboratory specimens.

■ Heavy Machines Overdesigned

In the design of heavy machines and static structures, the shortcomings of our means for determining stress are overcome by overdesign, commonly considered as factors of safety. For low-volume production machines in which weight is relatively unimportant, this practice is well justified because the cost of designing to low stress tolerance would far exceed the gain from savings in material and weight. The case is very different when we consider machines in which weight is all-important, as in airplane engines or where weight and cost must both be considered as in automobiles and other mass-production items. In such machines it is necessary that each unit of material shall do the maximum of work consistent with reasonable cost, and it is in the design of such machines that the inadequacy of our means of stress determination and our ignorance of fatigue strength of materials are most keenly felt.

In the design of light machines where we cannot seek safety in oversize parts but must design as exactly as possible to the required strength, we frequently find that an experienced designer can estimate the required dimensions of a part more accurately than they can be calculated by textbook methods. Nevertheless, stress calculations must be made as first approximations in the design of parts for a new machine. Such calculations usually follow conventional methods but, instead of applying arbitrary factors of safety, we are guided by accumulated experience in the form of limiting stresses which vary with the kind of machine part that is being considered. For example, as rough measures, automobile crankshafts may be stressed, by calculation, to 20,000 psi; connecting rods will tolerate calculated stresses of 40,000 psi; valve springs do somewhat better, approximately 90,000 psi; disc clutch springs are good for 180,000 psi; while other forms of clutch springs may safely carry 600,000 psi calculated stress. These few examples are given to illustrate that calculated stresses are, in themselves, meaningless and are of value only when they are interpreted in terms of experience. At best, such calculations are little better than guesses and serve only as starting points. Each part must be tested thoroughly by all available means and modified as needed before it can be released for production which means only that our experience data are not, in most cases, sufficiently organized to permit the construction of reliable empirical formulas.

The accuracy of stress data from photo-elastic and extensometer readings is usually greater than can be obtained from the most involved mathematical analysis, but that they are far from reliable can be shown easily by fatigue tests. Two specimens of identical material, heat-treatment, and dimensions will show identical stress when measured by photo-elasticity or by extensometer; yet these specimens may vary widely in fatigue strength, depending upon minute differences in surface finish or internal stresses. Since internal stresses are often unavoidable due to processing operations, such as machining, heat-treating, straightening, or grinding, and since surface finishes vary all the way from rough forgings to lapped or honed surfaces, there is little reason to expect accuracy from extensometer readings and even less for photo-elastic tests since these specimens

IT is in machines where weight is all-important, such as in airplane engines, or where weight and cost must both be considered, as in automobiles, that the inadequacy of our means of stress determination and our ignorance of fatigue strength of materials are most keenly felt, the author of this paper believes. Calculated stresses, he contends, are in themselves meaningless and are of value only when they are interpreted in terms of experience. Although conceding that the accuracy of stress data from photo-elastic and extensometer readings is usually greater than that obtained from the most involved mathematical analysis, he shows by fatigue tests that these methods are far from reliable.

In the major part of this paper, S-N diagrams

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[1] See *Metal Progress*, Vol. 32, February, 1941, pp. 202-206: "Improving Engine Axles and Piston Rods," by O. J. Horger and T. V. Buckwalter.

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must be free from internal stresses and must be made of another material.

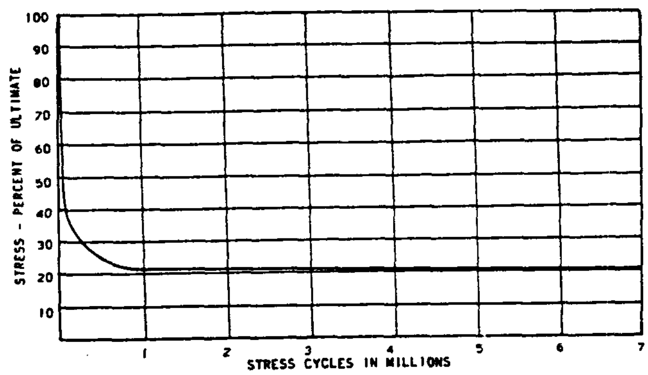
Photo-elastic and extensometer readings are measures of elasticity in which the changes in dimensions are the statistical average of all of the material involved in the measurement. Fatigue tests provide a strength measure of the weakest portion of the material involved, usually at the

with linear and logarithmic coordinates; combinations of these coordinates; and three-dimensional coordinates, are discussed. This discussion brings out the effect on fatigue strength of varying degrees of stress concentration; of surface treatment of the test specimens; of stress range; and presents considerable fatigue data on ball bearings. It is shown that, when fatigue-test results are run on a large number of commercially identical parts over a sufficiently large load range, the scatter of the test points, when plotted on logarithmic coordinates, falls within a well-defined pattern which tends to radiate from a point at high stress and low number of stress cycles and to diverge to a broad band at low stress and high number of stress cycles.

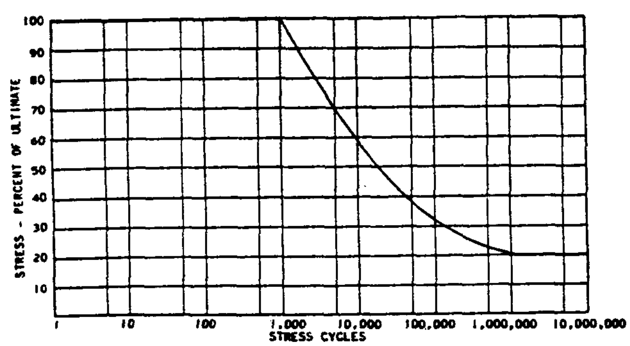
surface, even though it be sub-microscopic in size. Obviously, we cannot expect agreement between fatigue measures of stress and the stress readings obtained from elastic measurements.

This should not be construed to mean that stress-measuring devices should not be used. Photo-elastic tests are particularly useful in the classroom to demonstrate, qualitatively, the increase in stress at fillets, notches, holes and other section changes. Extensometer measurements have similar though somewhat broader utility since readings may be taken from actual machine parts to show the effect of section changes. These readings can sometimes point the way to alterations in design for more uniform stress distribution, providing that the manner of load application in service is known and not assumed, but they should be regarded as qualitative and not as quantitative measures of stress when applied to cyclically stressed parts.[1]

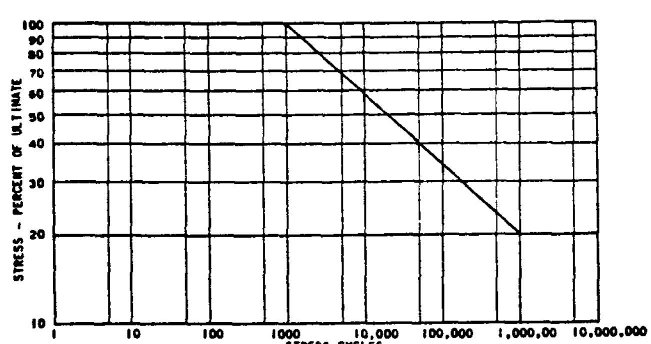
Fatigue test data as reported by many books and papers are not directly applicable to the design of light-weight, high-output machine parts because, with few exceptions, they assume: (1) that stress can be calculated; (2) that a machine part must be stressed below the fatigue endurance limit to be successful; (3) that laboratory test specimens are representative of a material when that material is used



■ Fig. 1 – Typical fatigue diagram plotted on linear coordinates



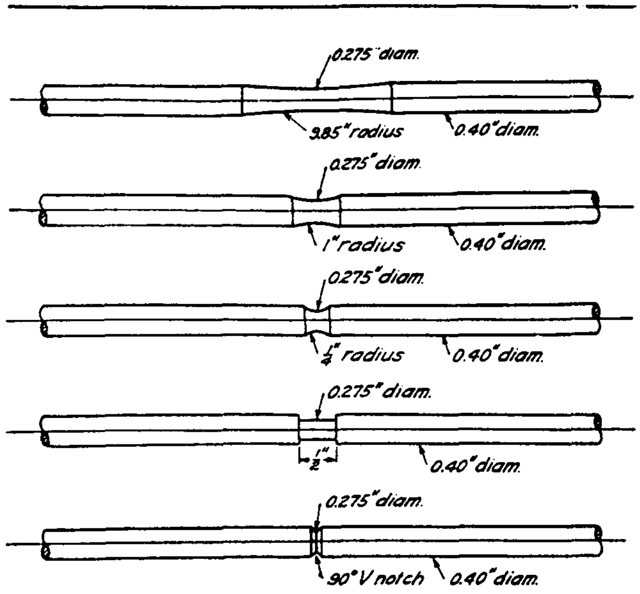
■ Fig. 2 – Typical fatigue diagram plotted on linear ordinates and logarithmic abscissa



■ Fig. 3 – Typical fatigue diagram plotted on logarithmic coordinates in a machine part; and (4) that a representative fatigue curve can be constructed from a dozen or less specimens.

Fatigue data are usually presented in graphs, called S-N diagrams, in which stress is plotted on the vertical axis and stress repetition or cycles are plotted on the horizontal axis. These graphs are variously constructed on linear coordinates, logarithmic coordinates or on linear ordinates and logarithmic abscissa. The linear coordinate plot shown in Fig. 1 is infrequently used in fatigue studies, notwithstanding the fact that it presents a clearer quantitative picture of the abrupt loss of strength of material when subjected to cyclic loads. As a working chart, however, this plot is inconvenient as will become apparent. The majority of fatigue workers prefer to use the linear ordinates and logarithmic abscissa scales since this arrangement contracts the N scale sufficiently to represent data over a large range of life. Fig. 2 shows the curve of Fig. 1 as it appears on such coordinates. The third form, less frequently used, is preferred for this paper because, as shown in Fig. 3, the fatigue curve of Figs. 1 and 2 appears substantially as a straight line when plotted on logarithmic coordinates. However, since there is no known way to determine true stress for machine parts, the stress scale in S-N diagrams can, at best, show only values proportional to stress. For specimens of varying shape, it is doubtful that the proportionality of the stress scale will be the same for any two specimen forms even though they are otherwise identical.

The assumption that machine parts must be stressed below the fatigue endurance limit has led most investigators in this field to test their specimens at stresses near this limit; that is, at stresses so low that the specimen will endure for an infinite number of stress repetitions without failure. There is, therefore, very little data from which to construct the characteristic curve of failure of specimens in the finite life region of fatigue – that is, that part of the diagram lying between one stress cycle and one million

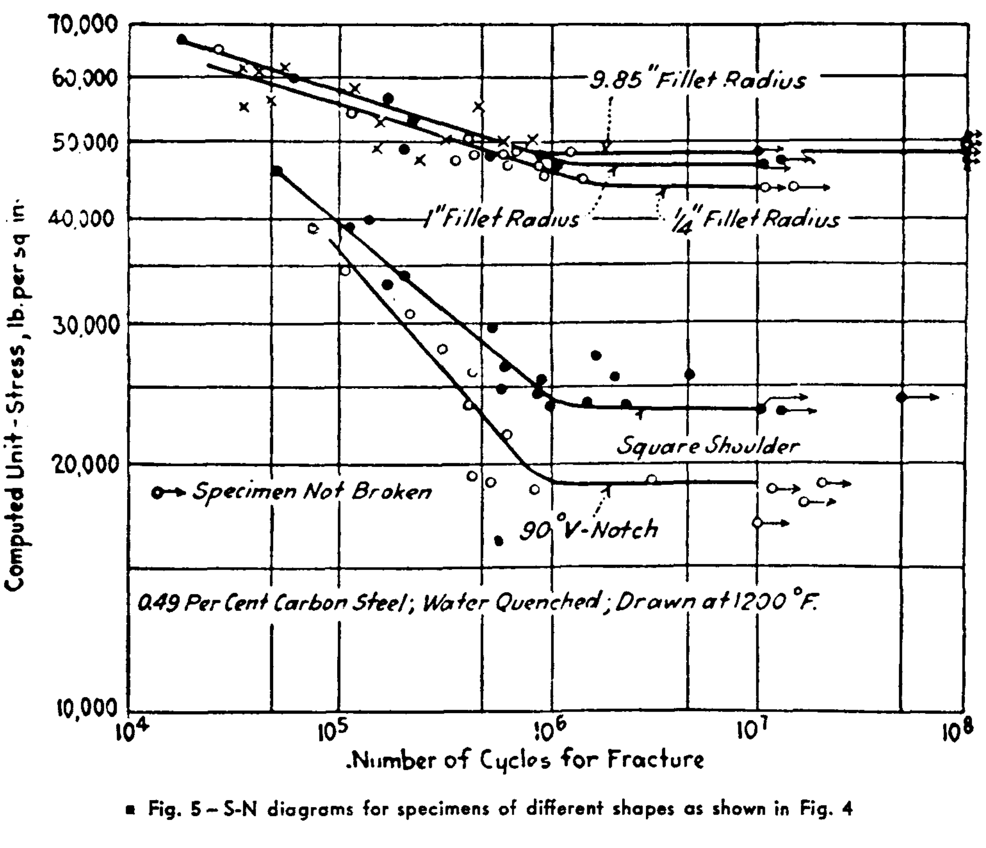


■ Fig. 4 – Specimens for study of effect of shape on endurance limit

stress cycles. Many machine parts are operated at maximum stresses above the fatigue endurance limit and failure, when failure occurs in such parts, is due to a relatively small number of stress cycles at maximum stress. Table 1 shows the life requirements at maximum stress of a few typical machine elements as determined by service experience.

These life requirements are fixed by the abusive type of user and are sufficiently high so that failure rarely occurs during the lifetime of the automobile except in the case of an accident. Obviously, to design such parts for infinite life at maximum stress would be economically wasteful.

The preferred laboratory fatigue test specimen is prepared very carefully to avoid all surface imperfections, abrupt section changes, internal stresses, and so on. This care is considered necessary because the investigator usually is interested in the properties of the material undergoing test and he naturally seeks to eliminate all factors that would tend to obscure these inherent properties. There can be no objection to this procedure as it refers to the test specimens, but the data thus obtained can have little bearing on the fatigue characteristics of machine parts made from the same material and given the same heat-treatment in which surface irregularities, abrupt changes in sec-



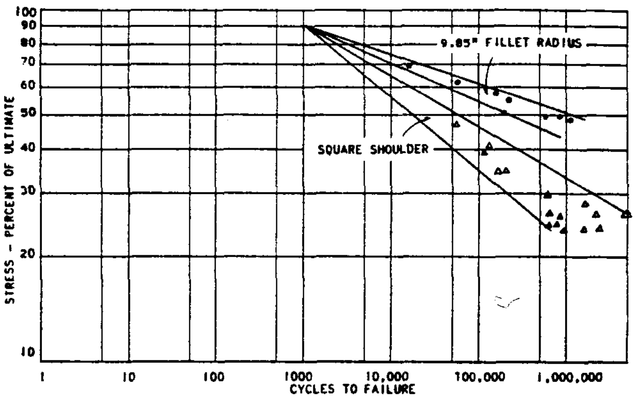
■ Fig. 5 – S-N diagrams for specimens of different shapes as shown in Fig. 4

Table 1 - Life Requirements at Maximum Stress in Cycles

Automobile rear-axle gears	100,000
Automobile transmission low gear	100,000
Auto chassis springs	100,000
Automobile transmission second gear	300,000
Truck rear axle	500,000
Bus rear axle	1,000,000

tion, and internal stresses are almost always present.

Many laboratory fatigue tests have been reported from specimens having varying degrees of surface roughness and various notch forms. These data are important in that they show very different characteristics from the smooth-surfaced uniform-section specimens. Fig. 4 shows a series of fatigue specimens used by Moore and Kommers[2] to determine the effect on fatigue of varying degrees of stress concentration. The resulting fatigue curves, plotted on logarithmic coordinates, are shown in Fig. 5. The authors



■ Fig. 6 – Probable form of scatter band of specimens 1 and 4, Fig. 4, if a large number of specimens had been tested

compare these specimens on the basis of calculated stress at the fatigue endurance limit; that is, the stress at the "knee" where the curve becomes horizontal. However, as stated before, our interest is in the finite life region of the diagram; that is, in the characteristic of the curve lying to the left of the "knee." Observe that, as the severity of the specimen section is increased the slope of the curve increases, and that the curves, if extended left-ward, tend to cross one another. Fatigue curves of machine parts, no matter how well finished or how carefully rejected for detectable flaws, almost invariably show steeper slopes than are shown by well-finished fatigue specimens and, therefore, presumably the fatigue strength of a material as determined by ideal test specimens is not obtainable when that material is formed into a machine part. Permissible stress at the fatigue limit of a machine part may be less than 10% of the ultimate strength of a material, whereas laboratory test specimens may indicate 50% or more as obtainable. The differences in slope of fatigue curves suggest that this characteristic promises a way whereby we may eventually greatly improve our accuracy in determining the strength of machine parts. This is now being done in rating the load capacity of ball bearings, roller bearings, and certain gears.

Published fatigue curves without exception are based on an insufficient number of test specimens. The lines plotted in Fig. 5 are intended to represent the averages for the specimens tested. Note the wide scatter of the test points and the increasing scatter of the points as the slope increases. Note also that, generally, the scatter decreases toward the left of the diagram. The significance of this scatter is not apparent in the diagram due to the limited number of test points, there being an average of only 12 failed tests for each type of specimen. Fatigue data should be regarded as mortality data, particularly in the finite life region of the diagram. We would have little confidence in life-insurance policies for which the premiums were based on mortality data from a dozen random individuals. Likewise, we cannot hope to establish the fatigue characteristics of specimens or of machine parts from a dozen odd tests.

The scatter of test points is due to unavoidable differences in test specimens no matter how carefully they are made and, since these differences constitute varying degrees of stress concentration, the fatigue line representing the poorest specimens should lie on a steeper slope than the fatigue line representing the best specimens. This is for the same reason that the average slopes of the specimens shown in Fig. 4 increase with the severity of the stress concentration as shown in Fig. 5. The test points for any group of specimens would, therefore, be expected to lie within a scatter band diverging from the region of high stress.

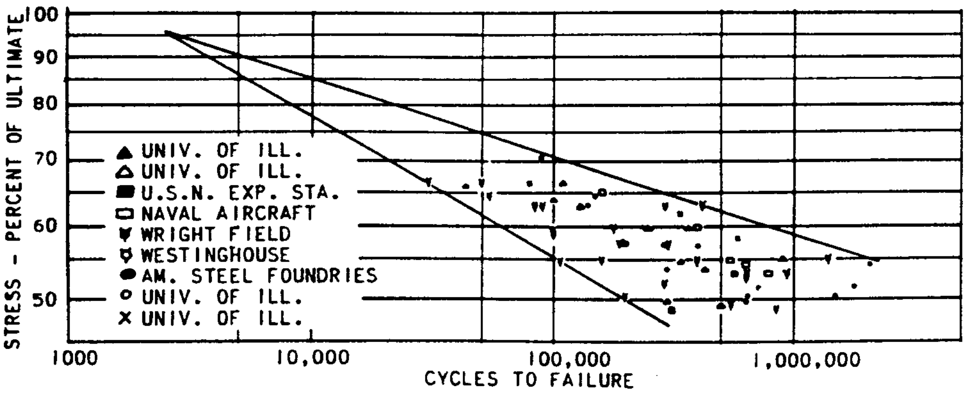
If a sufficient number of specimens had been tested, and if the stress scale proportionality were the same for all specimens, it is probable that the sloped lines in Fig. 5 would all tend to converge toward a point in the vicinity of 1000 cycles and 90,000 psi, somewhat as indicated in Fig. 6 in which the fatigue slopes of specimens 1 and 4 of Fig. 4 are shown as converging bands rather than lines. This region of intersection is suggested because the ultimate strength of the material tested by Moore and Kommers is approximately 95,000 psi and, obviously, if the stress scale is correct for each type of specimen, they will all have approximately the same strength at one stress cycle. The point of intersection would probably be at a considerable number of stress cycles because the ductility of the material permits adjustment of stress through yield, thus reducing the influence of local highly stressed points. For very brittle material, the intersection point of the fatigue curves for the type of specimens shown in Fig. 4 would probably be near the ultimate strength and nearer one cycle of stress.

There are not now available sufficient data on any specimens to complete a group of fatigue diagrams to the region of intersection. Knowledge of the characteristics of fatigue curves at high stress would be valuable in industry since it would facilitate greatly interpretation of fatigue tests on machine parts. Such tests could be evaluated in terms of the slope of the fatigue curve which would also give a clue to the actual stress, if desired, in the part being tested.

Fig. 7 shows an S-N diagram replotted from data contained in an ASTM Research Report[3]. By combining all the data for heat-treated SAE 4340 steel in a single S-N diagram, we have a total of 59 failed points which roughly outline a scatter band of the general form suggested in Fig. 6. This report also gives data on 54 failed points for

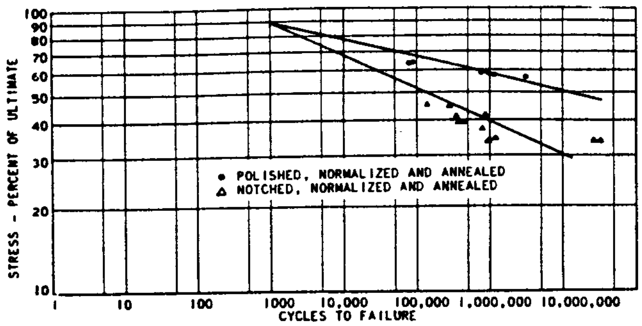
[2] See "Fatigue of Metals," by Moore and Kommers, McGraw-Hill Book Co.

[3] See Report of the Research Committee on the Fatigue of Metals, ASTM Meeting, June, 1941.



■ Fig. 7 – Re-plot of data reported by the ASTM Research Committee on Fatigue of Metals

■ Fig. 8 (below) – Fatigue properties of metals used in aircraft construction – 4134 steel



low-alloy steel (HT 50) which gives a less satisfactory scatter band since several points are reported as tested far above the ultimate strength of the material and the data are, therefore, doubtful.

Published data on fatigue of metals contain numerous tests showing the same general trend of increasing slope with increasing stress concentration, whether due to differences in specimen shape, specimen size, mechanical working of specimen surface, surface coatings, fillet radii, surface finish, or to variations between identical specimens. This tendency toward convergence is often not apparent in the published curves because the investigators have plotted their data on linear ordinates and logarithmic abscissa and always there are insufficient test points. The following diagrams copied from published papers have, when necessary, been replotted on logarithmic coordinates to the same scale as used by Moore and Kommers for the sake of uniformity, in which the stress scale is four times the scale of stress repetitions. The slopes of the curves are calculated as the measured horizontal distance multiplied by the scale ratio divided by the measured vertical distance. $\frac{(\text{Abscissa} \times \text{Scale Ratio})}{\text{Ordinate}}$ Strictly speaking, this value is the reciprocal of the slope, but it is used for convenience.

Oberg and Johnson[4] report a comparison between polished and notched specimens shown in Fig. 8 with results similar to the experiments by Moore and Kommers, Fig. 5.

Surface treatment of the test specimens other than the degree of smoothness has a marked effect on fatigue strength. Horger and Maulbetsch[5] compared normal well-finished specimens with specimens that had been subjected to a rolling operation which introduced compressive stresses in the surface layer with the results shown in Fig. 9. Since the rolled specimens were pre-stressed in compression, the subsequent tension stresses during the test were reduced; hence the difference in the slope of the curves for the two types of specimens. Since this treatment should be ineffective in a tensile test, the lines should converge as shown.

Swanger and France[6] made tests on zinc-coated specimens by which they demonstrated loss in strength due to the zinc coat. Fig. 10 shows the increase in slope of the curve for the coated specimens and the convergence of the curves toward the left.

[4] See ASTM Proceedings, Vol. 37, Part II, 1937, p. 199: "Fatigue Properties of Metals Used in Aircraft Construction at 3450 and 10,600 Cycles," by T. T. Oberg and J. B. Johnson.

[5] See ASME Transactions, Vol. 58, 1936, pp. A-91-A-98: "Increasing the Fatigue Strength of Press-Fitted Axle Assemblies by Surface Rolling," by O. J. Horger and J. L. Maultbetsch.

[6] See ASTM Proceedings, Vol. 32, Part II, 1932, p. 439: "Effect of Zinc Coatings on the Endurance Properties of Steel," by W. H. Swanger and R. D. France.

From the foregoing, it seems reasonable, as a working hypothesis, to assume that, except possibly for very ductile metals, the slope of the fatigue curve may be considered a measure of true stress and that the fatigue curves for varying stress concentrations converge toward a point near the tensile strength of the material and at some considerable number of stress cycles.

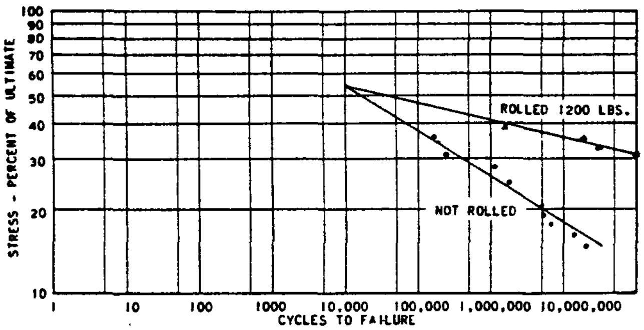
The application of this hypothesis to the fatigue strength of machine parts leads to some important implications. A large variety of machine elements are being tested constantly for relative durability in the laboratories of industries engaged in the manufacture of light-weight, high-output machines. In most cases, these fatigue tests are intended to compare one design, material, or process with another design, material, or process. It is axiomatic that nothing can be learned in regard to limiting loads except through tests to destruction. Therefore, the fatigue tests for practically all parts are run to failure and the comparison is made on the number of stress cycles at constant load that each part will withstand. This procedure is followed regardless of whether, in practice, the part in question is stressed below the fatigue limit or whether it is a part requiring relatively short life at maximum stress.

This method of evaluating test results is subject to serious error for several reasons. If it is true that fatigue curves radiate from a point in the high stress region, it is obvious that comparisons of specimens cannot be made on a percentage basis only, since the percentage difference will vary all the way from zero to infinity depending upon the load that is applied during the test. Furthermore, since the scatter band for each test part should also radiate from the same point as in Fig. 6, the width of the band in terms of life may be several hundred percent and, unless a considerable number of tests are run for each part, there is no assurance that whatever life difference is found is real or just the chance location of those particular test points within the scatter band. It is easily possible that the better design, material, or process will apparently rate lower than the poorer design, material, or process if an insufficient number of tests are made.

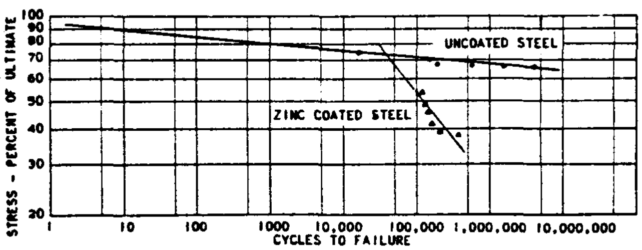
It is possible that the average fatigue curves for two materials having different tensile strengths and yield points will cross at some point in the finite life region due to differences in sensitivity to stress raisers. In such cases, life comparisons may be positive for one material at one test load and negative for the same material at another test load. It is evident, therefore, that true comparisons can be obtained only through fatigue tests on a sufficient number of parts at varying loads to outline the slopes of the scatter band limits. While this may appear to be an impractical requirement, it is not so difficult as it seems. It is only necessary that the results of the present routine tests be accumulated on a fatigue diagram and, in a relatively short time, fatigue curves and their scatter bands will be available for a large variety of machine parts.

Only occasionally are fatigue tests on machine parts run at various loads but, in the very few cases where data from a reasonable number of such tests run at sufficiently large load range are available on commercially identical parts, a reasonable number being one or two hundred, we find that the scatter of the test points when plotted on logarithmic coordinates falls within a well-defined pattern which tends to radiate from a point at high stress and a low number of stress cycles and to diverge to a broad band at low stress and high number of stress cycles just as was suggested in Fig. 6. This pattern is clearly shown in Fig. 11 which is a fatigue diagram of about 200 complete automobile and truck rear axles of various makes and sizes. The stress scale shown in this diagram is not true stress but is believed to be proportional to true stress. The axles were tested at loads to produce failure of one or more pinion teeth through the range of from 7000 cycles to 1,000,000 cycles. The scatter of the test points is due to variations in one or more of the many variables that are always present in commercially similar parts, such as internal stresses, fillet radii, cutter scratches, bearing, shaft and housing deflections, warpage in heat-treatment, and so on. The slope of the average durability line, calculated as the horizontal distance divided by the vertical distance measured on logarithmic coordinates is approximately 7, while the slopes of the upper and lower limits of the scatter band are, respectively, 9 and 5. The intersection point at the left of the diagram should lie near the ultimate strength of the material (approximately 300,000 psi) which, if proved, would supply us with a true measure of stress for the entire diagram. The diagram of Fig. 11 is not ideal as a proof of the scatter band or the intersection point, since it includes a variety of axles made from various alloy steels variously heat-treated for which the stresses were calculated by an empirical formula.

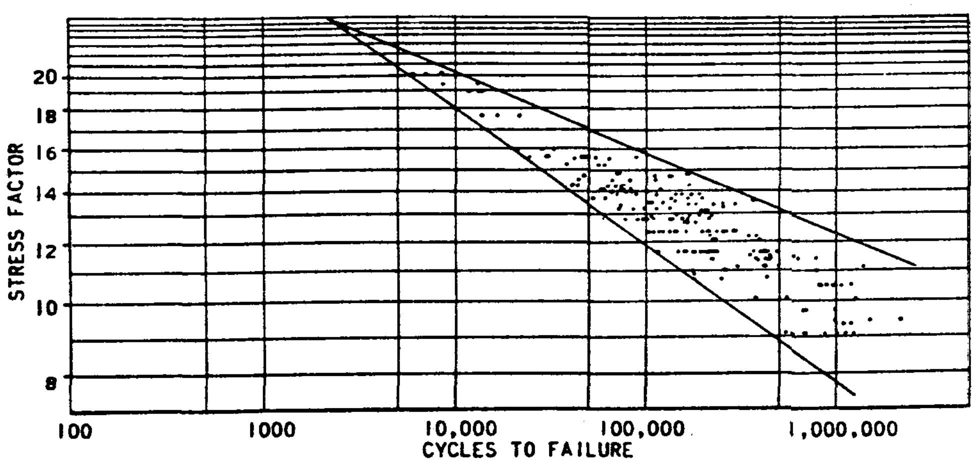
Satisfactory determination of the characteristics of the scatter band would require a large number of fatigue tests on one form and size of specimen, made of one type of material similarly heat-treated and tested to produce failure over a range of stress repetitions from as near a single cycle of stress as possible to the fatigue limit. Data approaching these requirements have been accumulated by the various ball and roller bearing manufacturers but very little has been published. However, fatigue data on ball and roller bearings need not in all particulars agree with fatigue data on other forms of machine parts since failure of roller bearings originates below the surface of the material. Surface influences, which play so important a part in fatigue of ordinary machine parts are, therefore, absent in roller bearings which would be expected to influence the permissible stress and possibly the form of the scatter band. The scatter band as reported by Macauley[7] and by Styri[8]



■ Fig. 9 – Fatigue test data showing the change of slope due to rolling



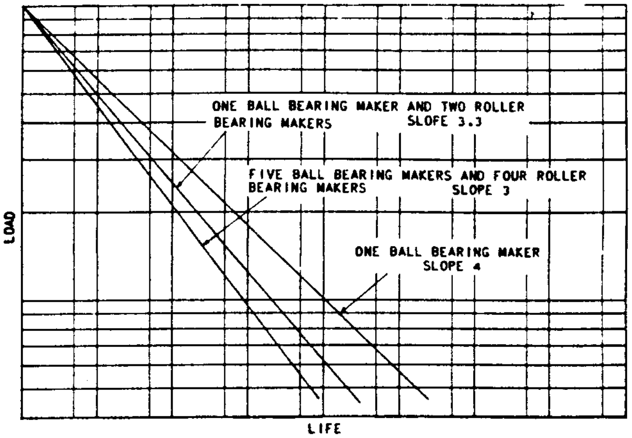
■ Fig. 10 (above) – Effect of zinc coating on fatigue strength



■ Fig. 11 – Scatter band for fatigue tests of 196 spiral-bevel rear-axle gears

[7] See *The Automobile Engineer,* Vol. 13, July, 1923, pp. 213-223: "The Endurance of Ball Bearings," by A. W. Macauley.

[8] See *Mechanical Engineering,* Vol. 47, June, 1925, pp. 490-492: "General Properties of Ball Bearings," by Haakon Styri.



■ **Fig. 12 – Characteristic fatigue curve slopes, load versus life, as obtained from catalogs of ball and roller bearing manufacturers – In this diagram only the slopes are significant**

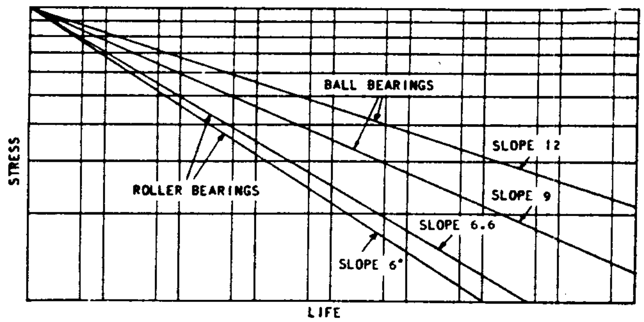
is parallel to the average life curve throughout the life range shown. The catalogs of ball and roller bearing manufacturers contain life and load data from which fatigue diagrams may be constructed but, although these diagrams reveal some very interesting facts, they do not supply us with scatter bands or show whether a logarithmic fatigue curve is, substantially, a straight line up to the ultimate strength of the material from which the bearings are made. Fig. 12 is a logarithmic diagram of load (not stress) versus life taken from the most recent catalogs of a number of leading ball and roller bearing manufacturers. All the lines, representing the various makes of bearings, are arbitrarily drawn from a point at the left of this diagram since we are now only interested in their slopes to show the similarity of the experience for all makes of bearings. The slopes for these lines are 3, 3.33, and 4.

It has been shown by Hertz and others that the stress in ball bearings varies as the cube root of the applied load and that in roller bearings the stress varies as the square root of the applied load. When the diagram of Fig. 12 is plotted in the usual terms, stress versus life, as in Fig. 13, we find that the slopes for ball bearings are 9 and 12, while for roller bearings the slopes are 6 and 6.66. The steeper slope for roller bearings presumably is the result of increased stress at the end of the roller due to the discontinuity of shape which thus constitutes a region of stress concentration. This further confirms the hypothesis that slope is a measure of stress.

Fig. 14 is a diagram of catalog rated load versus catalog rated life in terms of inner race revolutions for several makes of ball bearings of the same size races and the same size and number of balls. Note again the similarity of rated capacity for all bearings, the greatest difference being that one manufacturer rates his bearings on a fatigue slope of 12 in the 1941 catalog. In previous catalogs this manufacturer rated his bearings on a stress fatigue slope of 9 from which we may assume that he has either improved the durability of his bearings or that he has elected to rate his bearings on a new average line within the scatter band. The diagram, Fig. 14, is also interesting in that it does not show a fatigue limit[8] as is usually found in fatigue specimens. According to the catalog ratings, the sloped lines continue to more than a billion inner race revolutions and, since there are several stress cycles per revolution, we do not find a "knee" in these curves up to more than five billion stress cycles.

We seek to determine true stress only as a step in predicting the adequacy or inadequacy of our designs. Any other means that will enable us to predict the performance of our designs will do as well. Ball and roller bearing manufacturers do not consider stress at all in their catalog ratings, but rely entirely upon tabulated load capacities as determined by service experience correlated with laboratory test data on complete bearings. In practice, we are not only unable to calculate or to measure stress, but we do not even know the manner of load applications in service on the majority of machine parts.

The somewhat common belief that we can conduct reliable tests in the laboratory by reproducing the conditions of service is wholly erroneous. By the time the laboratory investigator has provided for all of the conditions that occur in service, he will, in the case of automobile parts, find himself on the road with a complete automobile and, even, then, he will not represent the type of driver who most severely taxes the strength of the machine. Many laboratory tests have been used, and are still being used, by which to grade materials and processes that are now known to have been very costly to the automobile and other industries. Thus, the fiction that a carburized part should have a hard case to resist wear and a tough core to resist breakage arose from laboratory im-



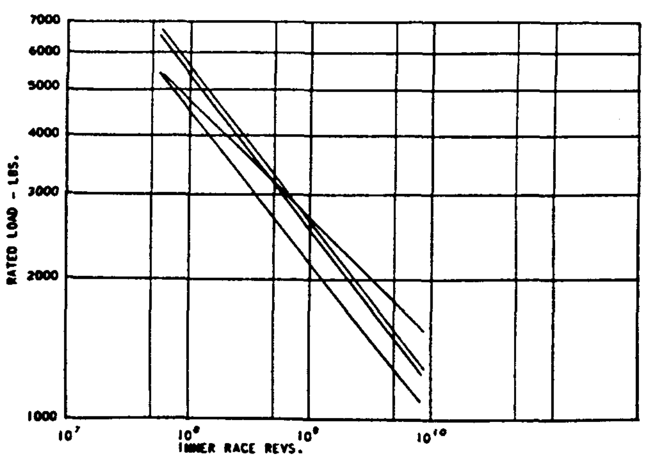
■ **Fig. 13 – Characteristic fatigue curve slopes, stress versus life – These are the same curves as shown in Fig. 12 except that load is converted into stress according to Hertz equations**

pact tests. In this test, the strength of the part was judged by the number of hammer blows a part would withstand before fracture and since, for example, gear teeth resisted impact fracture in accordance with the physical properties of the core, it seemed logical to specify heat-treatments to bring out the best compromise between the imagined requirements of the case and the core. Being compromises, these heat-treatments were not the best for either region. If, instead of counting the number of hammer blows to produce fracture, the gear tooth had been examined after the first impact, the tooth would have been found bent and, therefore, ruined. Hence it would make no difference how many more blows were required to fracture the tooth. This compromise heat-treatment resulted in reducing the quality of many millions of gears before it was realized that gear teeth fail by fatigue and that fatigue failure, for the usual depth of carburization, always originates at the surface of the case. It then became clear that the heat-treatment should consider the requirements of the carburized case only and that the properties of the core were relatively unimportant since the core serves mainly as a stuffing for the case.

Similarly, gear steels and steels for many other parts have long been selected by false standards that are based upon laboratory tests, among which are fatigue tests of ideal specimens. For many years, industry has paid premium prices for alloy steels because of their fancied advantages when used in gears and in other parts. Fatigue tests on actual machine parts correlated with service records have shown that there is no detectable difference between the high-priced alloy steels and the low-priced alloy steels when used in many machine elements. This result is probably due to the fact that, as fatigue specimens, machine parts are so far removed from the ideal laboratory fatigue specimen that the latter is misleading as a measure of worth. This is indicated by the relative slopes of the fatigue curves since, as already stated, the slope of fatigue curves for machine parts is always steeper than the slope of fatigue curves for the preferred laboratory specimen by which a material usually is judged.

Efforts to improve products by improving surface finish may sometimes have the opposite effect. Highly finished fillets may lead to a false sense of security if they are applied to parts having high internal stresses resulting from manufacturing operations or, as in ground gear teeth, the grinding operations may introduce high surface stresses in tension and thus promote fatigue failures. It is the writer's opinion that, from the standpoint of strength, more harm than good results from the grinding of gears. The surface stresses from grinding are often so great as to produce visible or magnaflux surface cracks but, whether detectable or not, surface tension is frequently very serious. Since fatigue cracks start on the side of the gear tooth that is loaded in tension, the effective stress is the grinding pre-stress plus the working stress. Laboratory fatigue testing of automobile or other light-weight, high-output machine parts, as well as other laboratory tests such as on fuels, oils, tire wear, and so on, must be correlated definitely with service data on the part in question before the results can be accepted. This requires that, for fatigue, tests must be devised that will agree with failures that occur in normal service as to the location of points of fracture and the character of the fractures whether or not the test procedures agree with preconceived notions of service loading.

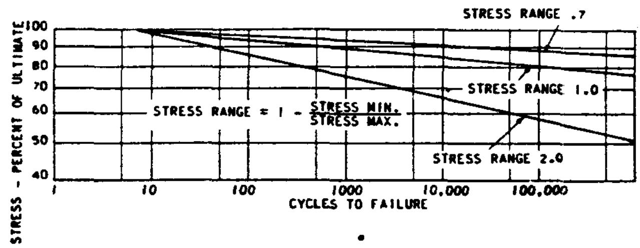
The slope of the finite life portions of the S-N diagram has been discussed from the standpoint of fatigue tests at constant stress range. Most of the test data presented hereto have been taken from specimens in which the stress was completely reversed. However, many machine parts



■ **Fig. 14 – Catalog rated load versus catalog rated life of four makes of ball bearings, all of same size, to show differences in load ratings of the different makers**

are otherwise stressed as, for example, valve springs which are stressed through a relatively narrow range in one direction only. Valve springs are preloaded to approximately 25,000 psi stress which is increased to approximately 90,000 psi when the valve is fully open. Gear teeth usually are loaded from zero stress to a maximum stress in one direction only. Axle shaft stresses are somewhat more complex being completely reversed in bending during each revolution due to the weight of the car while transmitting torque in one direction only.

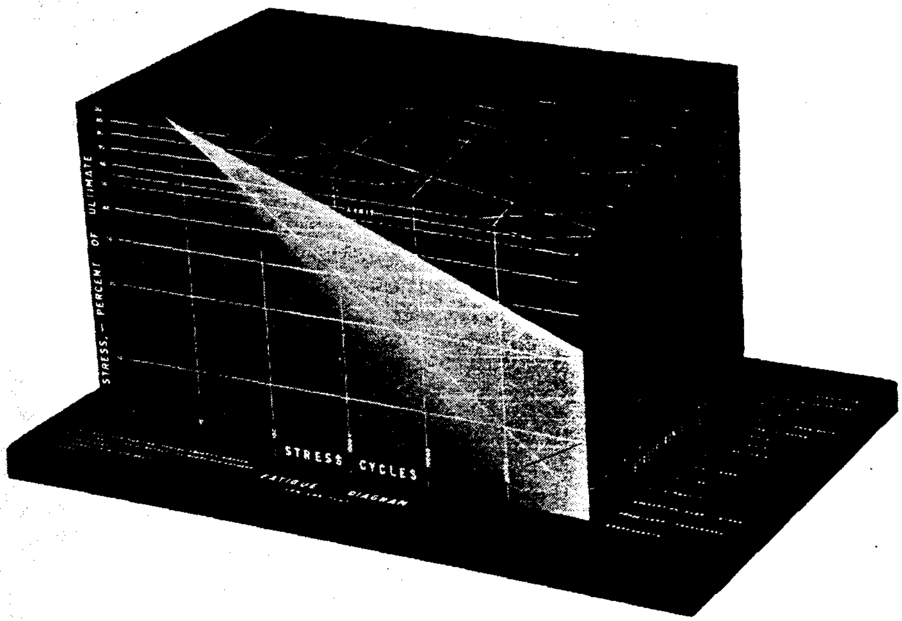
Many experiments have been conducted to determine



■ **Fig. 15 – Effect of stress range on fatigue life – The slope of fatigue curves varies with stress range as well as with stress concentration**

the effect of varying the stress range, but again interest lay in the stress at the fatigue limit and little data are available on the change of slope with stress range. However, since the stress at the fatigue limit increases as the stress range decreases as has been amply demonstrated, it follows that the slope must decrease (become flatter) as the stress range decreases. Moore and Kommers[2] present, somewhat apologetically, a modified Goodman diagram from which the fatigue slope for ideal specimens may be constructed for any stress range. Fig. 15 shows a re-plot of this modified Goodman diagram for three stress ranges. The upper curve represents a small stress range similar to that of automobile valve springs; the second curve represents a stress range of from zero to maximum as for gears; and the lowest curve represents complete reversal of stress as may occur in a crankshaft. The slopes of these curves are respectively 80, 48, and 17. We thus see that the slope of the fatigue curve varies with stress range as well as with stress concentration and, therefore, the hypothesis that the slope of the fatigue curve is a function of stress is no longer tenable. If, however, we state that the slope of the fatigue curve is a function of effective stress, the hypothesis will apply for any stress range.

Fig. 16 is a three-dimensional diagram on logarithmic coordinates of the modified Goodman diagram by Moore and Kommers. The vertical scale of this diagram represents stress, the horizontal scale on the forward side represents stress cycles, and the horizontal depth scale represents stress range. The forward face is the ordinary S-N diagram for complete reversal of stress; the back face is an S-N diagram for a stress range of zero; and a section at the middle of the stress-range scale would be an S-N diagram for a stress range of zero stress to maximum. The numerical values of the stress-range scale have been arbitrarily selected so that 2 represents complete reversal of stress; 1 represents stress from zero to maximum; and zero represents no change in stress. The equation $R = 1 - \frac{\text{minimum stress}}{\text{maximum stress}}$ has, therefore, been used by

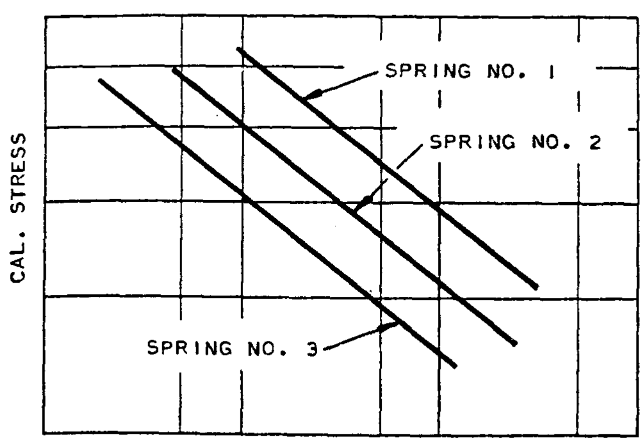


■ Fig. 16 – Three-dimensional fatigue diagram showing the effect of stress range on the slope of the fatigue curve

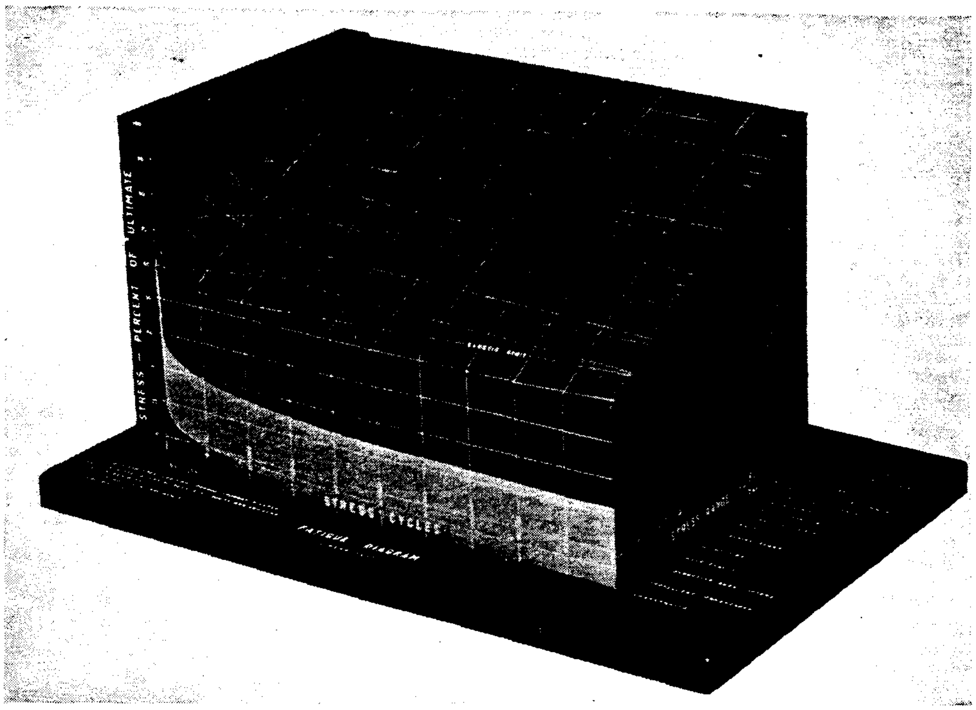
which to locate points of intermediate stress range. On the forward face of this diagram are drawn two straight lines diverging from a point at the upper left. These two lines represent approximately the best slope (upper) and the poorest (lower) that have been obtained from available fatigue tests on machine parts at complete stress reversal. These slopes are such that, at one million stress reversals, the poorest specimens (slope 4) are stressed, by hypothesis, between 7 and 8% of the tensile strength of the material, the best specimens (slope 8) are stressed, by hypothesis, to about 23% of the tensile strength, whereas the slope of the Moore and Kommers' diagram (slope 17) is stressed to 50% of the tensile strength. These conclusions are based upon the further hypothesis, previously stated, that fatigue diagrams are straight lines, on logarithmic coordinates, to a point on the tensile strength line at some considerable number of stress cycles. Since the data on machine elements indicating the slopes of their fatigue curves are meager, the diagram should be considered as qualitative only. The stress-range (depth) face also shows lines that are even less supported by reliable evidence. They also represent the best and the poorest of fatigue tests on machine parts and are based on one point each at stress range 2, at stress range 1 and converging to a point at stress range zero and ultimate stress. The diagram is presented here in the hope that other experimenters will come forward with data to prove or disprove the hypothesis upon which it is based.

Fig. 17 is the same diagram as Fig. 16 except that it is plotted on linear coordinates better to present dimensional values, particularly to those who are not accustomed to the use of logarithmic charts.

Since we have no reliable means for determining stress and since fatigue tests on laboratory specimens cannot be used for evaluating the strength of machine parts, we have no recourse but to continue fatigue tests on machine parts in our industrial laboratories. There is, however, much that we can do to improve our technique in setting up the conditions of tests and in interpreting the test data. The methods now used for coordinating laboratory tests with service are too haphazard to be completely reliable. Service failures must, obviously, be infrequent and when true fatigue failure does occur, it is the result of harder-than-usual service combined with a point lying on the lower fringe of the fatigue scatter band. Since failures must be infrequent, it is highly important that failed parts be examined by competent observers in order that the true cause of the trouble may be determined. Clear evidence of fatigue failure does not prove that the failed part was primarily responsible. A bolt may fatigue because it was not properly tightened during assembly; a gear may fatigue due to improper support or to a failed bearing; a crankshaft may fatigue due to inadequate or maladjusted vibration damper; and so on without end. It sometimes happens, therefore, that immediate corrections are made to the wrong part and recognition of the true trouble is sometimes greatly delayed. Laboratory fatigue tests on machine parts must not only duplicate service failure as to location of fracture, but they must, in many cases, produce failure in approximately the same number of stress cycles if accurate life comparisons are to be made. This requires that

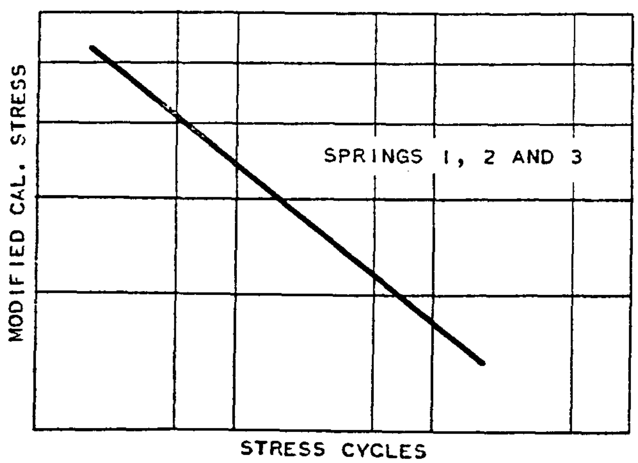


■ Fig. 18 – Inadequate formula



■ Fig. 17 – Same as Fig. 16 except that linear coordinates are used

we distinguish between normal operation stress and the relatively infrequent overloads that caused the failure. Rear-axle automobile gears are, at worst, stressed at low-gear torque one cycle out of every one thousand cycles. The lifetime requirements of such gears, therefore, is 100,000 cycles at low-gear torque equal to approximately 30 miles of travel. Due to the scatter of test points this is approximately 250,000 cycles measured on the average fatigue curve. Chassis springs normally operate through a small stress range, but they must be designed to withstand a total of high stress cycles equal to the number of bumps that will be experienced by the hardest driver on the worst road. This is a relatively small number of stress cycles but the problem is aggravated by the fact that such springs are subjected to severe corrosion and to surface damage by stones. Hence experience requires an average lifetime of 100,000 cycles at maximum amplitude. Clutch springs in town driving are deflected approximately 500,000 times during the lifetime of an automobile, but not always at the maximum amplitude. Therefore, an average life of 500,000 cycles at full amplitude is a minimum requirement.



■ Fig. 19 – Modified formula

Fatigue data from machine parts can be studied profitably for the purpose of constructing empirical formulas by which load capacity eventually may be accurately calculated. Let us assume that we have accumulated fatigue data on one hundred coil springs each of three quite different sizes. We calculate the stress and plot these points on a logarithmic chart, as shown in Fig. 18, and we find that all springs of one size lie within a definite scatter band; all test points of the second size spring also lie within a definite scatter band having the same mean slope as the first group but somewhat lower on the chart; and the third set of springs plot like the other two but at still another level. From this plot we may conclude immediately that the formula used for calculating stress is in error since, by its use, we were unable to predict relative load capacity. If we now introduce into the formula a suitable size factor such as to bring the three curves and their scatter bands into coincidence, as shown in Fig. 19, we will have improved the accuracy of our calculations. This is the method that has been followed in arriving at the load capacities of ball and roller bearings and for automobile gears. By this procedure we may construct accurate load-capacity formulas for any machine part no matter how complex the stress pattern may be. A large number of fatigue test points are required, but this need not be a serious obstacle. Fatigue tests have been made for many years by many laboratories and there are, therefore, now sufficient data, if they can be assembled, to construct formulas for a considerable variety of machine parts, with more data being accumulated every year. The stress calculated by such formulas will not be true stress but this is unimportant because we will not have reliable means by which to determine load capacity. If, at some future time, it can be shown that the hypothetical fatigue curve, Fig. 6, or some equivalent system, applies to structural materials, it will become possible to determine true stress. It is suggested that this latter phase of the problem can best be entrusted to the laboratories of our technical schools.