Prior portions of Otto Almen's notable talk before ⚙ Chapters on "Fatigue of Metals as Influenced by Design and Internal Stresses" have described some common service failures and means whereby the situation can be improved (February, page 209, and May, page 737). Last month the proposition was argued that engineers in mass production industries should usually design for a limited life, and that installment and the present show how the sloping part of an endurance curve then becomes more useful than the stress for infinite endurance.

ENDURANCE OF MACHINES UNDER A FEW HEAVY LOADS

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IN the article on page 254 of last month's issue, entitled "The Useful Data to Be Derived From Fatigue Tests", a working hypothesis was established that the *slope* of the fatigue curve of parts made of heat treated steel, as measured on a log-log plot, may be considered a measure of effective stress. That is, if the line is flat, the effective stress is close to the amount computed for ideal shape and surface; if the line is steep, the test pieces contain stress raisers, either due to irregular surface contour or metallurgical irregularities. Fatigue curves for varying degrees of resulting stress concentrations converge toward a point near the tensile strength of the material, and located out at some considerable number of stress cycles.

It will now be shown how this hypothesis has been used.

It should be remembered that the tensile strength in a test in which the load is slowly increased is lower than in a tensile test in which the load is maintained for a very short time, as in a fatigue test, and also that there is a considerable variation in the tensile strength of any material as measured by a number of tensile test specimens. (See "Deformation and Fracture of Mild Steel Under Cyclic Stresses in Relation to Crystalline Structure" by Gough and Wood, Institution of Mechanical Engineers, March-October 1939, pag[illegible]) Therefore, the tensile strength on a fatigue chart would actually plot as a band and not as a line and would lie above the normal tensile value. Likewise, the lines of a fatigue plot would converge to a region above the normal tensile strength and would probably not meet at a point. However, the inclusion of these variables would considerably complicate the above hypothesis and since they occur in a region of the fatigue plot that has little or no practical value, they may, for the present, be ignored.

The application of this hypothesis to the fatigue strength of machine parts has some important implications. A large variety of machine elements are constantly being tested for relative durability in the laboratories of industries engaged in the manufacture of light weight, high output machines. In most cases these fatigue tests are intended to compare one design, material, or process with another design, material, or process.

Destructive Tests Necessary

It is axiomatic that nothing can be learned in regard to limiting loads except through tests to destruction and, therefore, the fatigue tests for practically all parts are run to failure and the comparison is made on the number of stress cycles at constant load that each part will withstand. As stated above, this procedure is followed regardless of whether, in practice, the part in question is stressed below the fatigue limit or whether it is a part requiring relatively short life at maximum stress.

This method of evaluating test results is subject to serious error for several reasons. If it is true that fatigue curves radiate from a point in the high stress region, it is obvious that comparisons of specimens cannot be made on a percentage basis only, since the percentage difference will vary all the way from zero to infinity depending upon the load that is applied during the test.

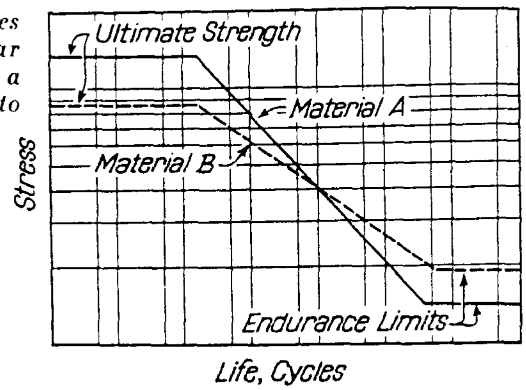


Fig. 11 — Average Fatigue Curves of Two Materials of Similar Strength May Cross if One Has a Higher Ratio (Endurance Limit to Ultimate Strength)

Furthermore, since the scatter band existing for each machine part tested should also radiate from the same point, as was shown in Fig. 4 of last month's article, the width of the band in terms of life may be several hundred per cent, and unless a considerable number of tests is run for each part there is no assurance that whatever life difference is found is real or just the chance location of these particular test points within the scatter band. It is easily possible that a better design, material, or process will apparently rate lower than the poorer design, material, or process if an insufficient number of tests is made.

Fatigue Curves May Cross

It is possible that the average fatigue curves for two materials having different tensile strengths an dyield points will cross at some point in the finite life region due to differences in sensitivity to stress raisers. In such cases, life comparisons may be positive for one material at one test load and negative for the same material at another test load, if the two are put into competition. The diagram on this page, Fig. 11, illustrates such a situation.

It is evident, therefore, that true comparisons can only be obtained through fatigue tests on a sufficient number of parts at varying loads to outline the slopes of the scatter band limits. While this may appear to be an impractical requirement, it is not so difficult as it seems. It is only necessary that the results of the present routine tests be accumulated on a fatigue diagram, and in a relatively shotr time fatigue curves and their scatter bands will be available for a large variety of machine parts.

Only occasionally are fatigue tests on machine parts run at various loads. In the very few cases where data from a reasonable number of such tests run at sufficiently large load differences are available on commercially identical parts, a reasonable number being 100 or 200, we find that the scatter of the test points when plotted on logarithmic coordinates falls within a well-defined pattern. This pattern tends to radiate from a point at high stress and a low number of stress cycles, and to diverge to a broad band at low stress and high number of stress cycles, just as was suggested in Fig. 4 of last month's installment.

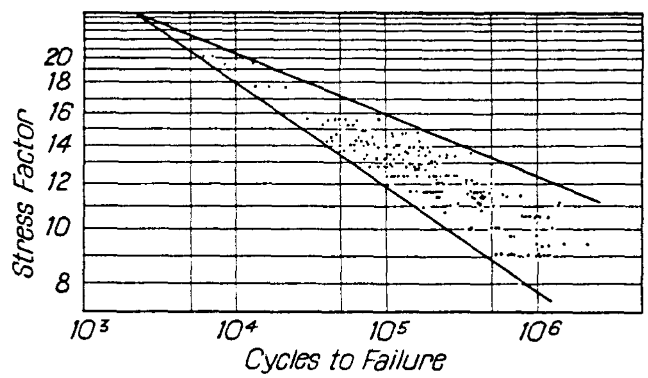


Fig. 12 — 196 Tests to Destruction on Rear Axles for Automobiles (Almen and Boegehold)

Endurance of Rear Axle Gears

This is clearly shown in Fig. 12, which is a fatigue diagram of about 200 complete automobile and truck rear axles of various makes and sizes, taken from a paper by Almen and Boegehold in *Proceedings*, American Society for Testing Materials, Vol. 35, Part 2, 1935, entitled "Rear Axle Gears — Factors Which Influence Their Life". The stress scale shown in this diagram is not actual stress but is believed to be proportional to actual stress. The axles were tested at loads to produce fatigue of one or more pinion teeth through the range of from 7000 cycles to 1,000,000 cycles. The scatter of the test points is due to variations always present in commercially similar parts, such as in residual stresses, fillet radii, cutter scratches, bearing, shaft and housing deflections, and warpage in heat treatment.

The slope of the average durability line, calculated as the horizontal distance divided by the vertical distance measured on logarithmic coordinates is approximately seven, while the slopes of the upper and lower limits of the scatter band are respectively nine and five. The intersection point at the left of the diagram should lie near the ultimate strength of the material (approximately 300,000 psi.) which, if proved, would supply us with a measure of actual stress for the entire diagram.

The diagram is not ideal as a proof of the scatter band or the intersection point, since it includes a variety of axles made from various alloy steels variously heat treated for which the stresses were calculated by an empirical formula. Satisfactory determination of the characteristics of the scatter band would require a large number of fatigue tests on one form and size of specimen, made of one type of material similarly heat treated, and tested to produce failure over a range of stress repetitions from as near a single cycle of stress as possible up to the number required to set a true endurance limit.

Ball and Roller Bearing Tests

According to *The Ball Bearing Journal*, No. 3, 1937 (SKF), data approaching these requirements have been accumulated by the various ball and roller bearing manufacturers, but the published data are not yet extensive enough to define the form of the scatter band. Particularly, more data are needed in the very low stress range and in the very high stress range. However, fatigue data on ball and roller bearings need not in all particulars agree with fatigue data on other forms of machine parts, since failure of rolling bearings usually originates below the surface of the material. Many surface influences, which play important parts in fatigue of ordinary machine parts, are, therefore, absent in rolling bearings. This would be expected to influence the permissible stress and possibly the form of the scatter band. The scatter band as reported by Macauley (*The Automobile Engineer*, Vol. 13, July 1923, page 213) and by Styri (*Mechanical Engineering*, Vol. 47, June 1926, page 490), is parallel to the average life curve throughout the life range shown.

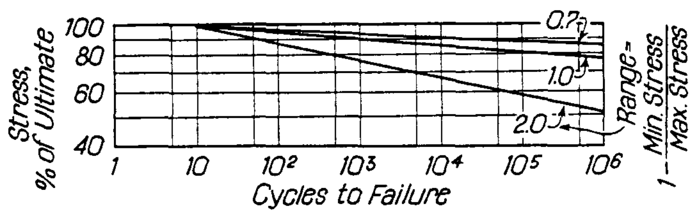


Fig. 13 — Slope of Fatigue Curve for Various Stress Ranges — Top as for a Valve Spring, Middle as for a Gear Tooth, Bottom as for a Crankshaft

Ball and roller bearings are also peculiar in that their S-N curves show no fatigue endurance limit as is usually found in steel fatigue specimens. According to the catalog ratings, the sloped lines continue to more than a billion (10^9) inner race revolutions and since there are several stress cycles per revolution, we do not find a knee in these curves up to more than five billion (5×10^9) stress cycles. See *The Ball Bearing Journal*, No. 3, 1937 (SKF, Toronto).

We seek to determine actual stress only as a step in predicting the adequacy or inadequacy of our designs. Any other means that will enable us to predict the performance of our designs will do as well. Bearing manufacturers do not consider stress in their ratings but rely entirely upon tabulated load capacities. These capacities have been determined by service experience, correlated with laboratory tests on complete bearings. In practice we are not only unable to calculate or to measure stress accurately, but we do not even know the manner of load applications in service on the majority of machine parts.

Laboratory fatigue testing of automobile or other light-weight, high-output machine parts, as well as other laboratory tests such as on fuels, oils, and tire wear, must be correlated with service data on the part in question before the results can be accepted. This requires that, for fatigue strength, tests must be devised that will agree with failures that occur in normal service as to the location of points of fracture and the character of the fractures — whether or not the test procedures agree with preconceived notions of service loading.

Effect of Stress Range

The slope of the finite life portions of the S-N diagram has now been discussed from the standpoint of fatigue tests at constant stress range. Most of the test data presented hereto have been taken from specimens in which the stress was completely reversed. However, many machine parts are otherwise stressed as, for example, properly tightened bolts in which the stress range approaches zero. Valve springs are stressed through a relatively narrow range in one direction only, being preloaded to approximately 25,000 psi. stress which is increased to approximately 90,000 psi. when the valve is fully open. Gear teeth usually are loaded from zero stress to a maximum stress in one direction only. Axle shaft stresses are somewhat more complex, being completely reversed in bending during each revolution due to the weight of the car; they also transmit torque in one direction only.

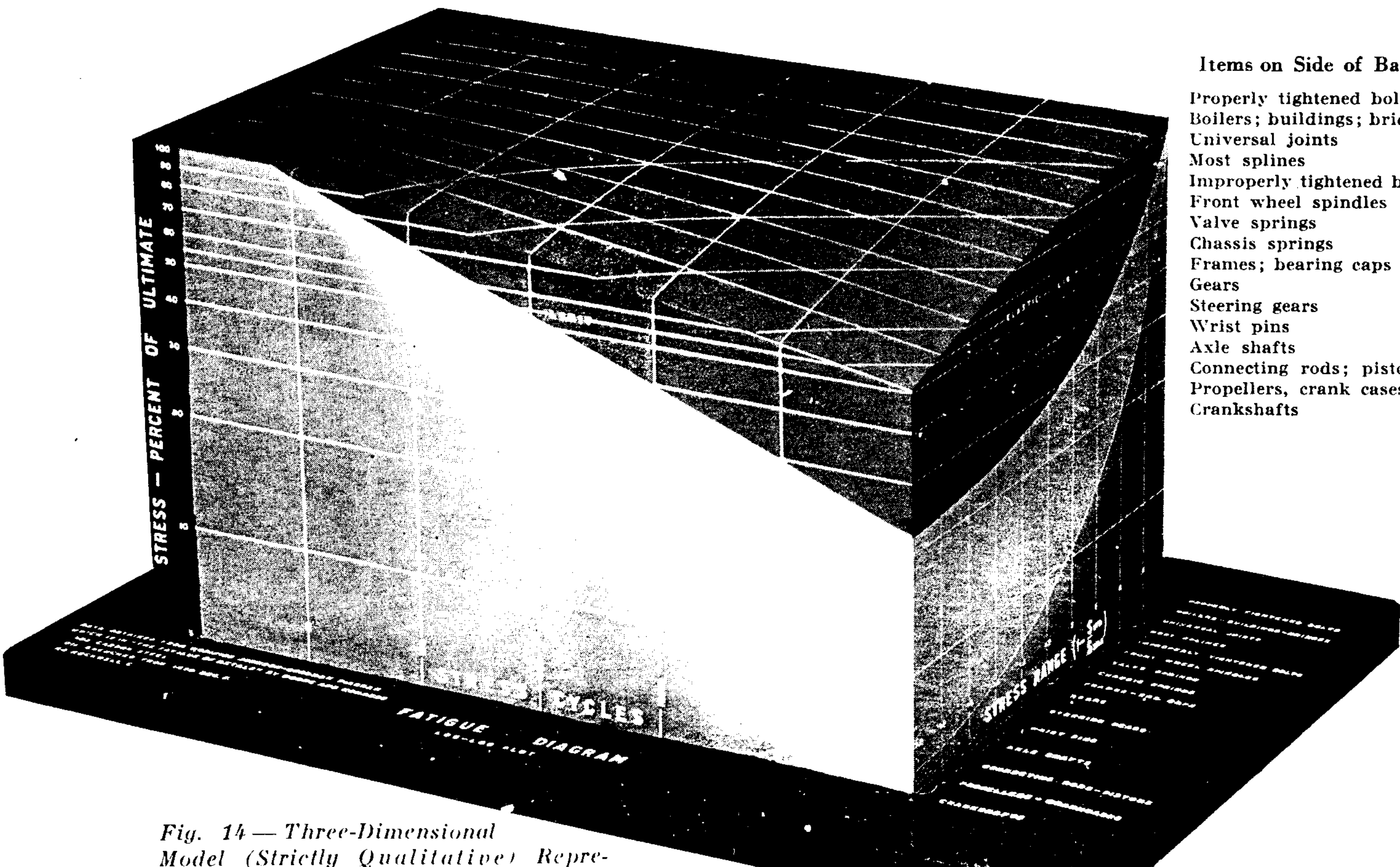
Experiments have been made to determine the effect of varying the stress range, but again interest lay in the stress at the fatigue limit, and little data are available on the change of slope of the curve with stress range. However, since the stress at the fatigue limit increases as the stress range decreases, as has been amply demonstrated, it follows that the slope must decrease (become flatter) as the stress range decreases.

Moore and Kommers in their book on "Fatigue of Metals" present, somewhat apologetically, a modified Goodman diagram from which the fatigue slope for ideal specimens may be constructed for any stress range. Figure 13 (p. 437) shows a replot of this modified Goodman diagram for three stress ranges. The upper line represents a small stress range similar to that of automobile valve springs; the second curve represents a stress range of from zero to maximum as for gears; and the lowest curve represents complete reversal of stress as may occur in a crankshaft. The slopes of these curves are respectively 80, 48 and 17.

We thus see that *slope* of the fatigue curve varies with stress range as well as with stress concentration and, therefore, the hypothesis that the slope of the fatigue curve is a function of the usual conception of stress is no longer tenable. If, however, we state that the slope of the fatigue curve is a function of *effective* stress, the hypothesis will apply for any stress range.

Three-Dimensional Fatigue Diagrams

Figure 14 is a three-dimensional diagram of the modified Goodman diagram by Moore and Kommers. The vertical logarithmic scale of this diagram represents stress, the horizontal logarithmic scale on the forward side represents stress cycles, and the arbitrary horizontal depth scale (front to back) represents stress range.



Items on Side of Base:

Properly tightened bolts
Boilers; buildings; bridges
Universal joints
Most splines
Improperly tightened bolts
Front wheel spindles
Valve springs
Chassis springs
Frames; bearing caps
Gears
Steering gears
Wrist pins
Axle shafts
Connecting rods; pistons
Propellers, crank cases
Crankshafts

Fig. 14 — Three-Dimensional Model (Strictly Qualitative) Representing on Its Front Face the Influence of Commercial Variations in Manufacture on the Effective Stress Causing Fatigue Failure at Relatively Short Life, and in Its Depth Representing Effect of Stress Range (Complete Reversal at Front; Slight Fluctuation at Back). At side is a suggested list of actual machine parts whose working stresses come within the indicated range

The forward face is the ordinary S-N diagram for complete reversal of stress; the back face is an S-N diagram for a stress range of zero (static stress); and a section at the middle of the stress-range scale would be an S-N diagram for a stress range of zero stress to maximum stress. The numerical values of the stress-range scale have been arbitrarily selected so that 2 represents complete reversal of stress, 1 represents stress from zero to maximum in tension, and zero represents no change in stress. The equation used in Fig. 13, page 437,

$$R = 1 - \frac{\text{minimum stress}}{\text{maximum stress}}$$

has, therefore, been used to locate points of intermediate stress range.

We find that on the light-colored forward face of this diagram are drawn two straight lines diverging from a point at the upper left. These two lines represent approximately the best slope (upper) and the poorest (lower) that have been obtained from available fatigue tests on machine parts at complete stress reversal. These slopes are such that, at one million stress reversals, the poorest specimens (slope 4) are capable of supporting, by hypothesis, between 7 and 8% of the tensile strength of the material; the best specimens (slope 8) are capable of supporting, by hypothesis, about 23% of the tensile strength of the material; whereas the slope of the Moore and Kommers' diagram (slope 17, the upper edge of the forward face) indicates that, at 1,000,000 reversals, the specimen should support about 50% of the tensile strength of the material.

These conclusions are based upon the further hypothesis, previously stated, that fatigue diagrams are straight lines, on logarithmic coordinates, reaching up and ending at a point on the tensile strength line at some considerable number of stress cycles. Since the data on machine elements indicating the slopes of their fatigue curves are meager, the diagram should be considered as qualitative only.

The stress-range (depth) face shows lines that are even less supported by reliable evidence. They also represent the best and the poorest of fatigue tests on machine parts and are based on one point each at stress range 2, at stress range 1 and converging to a point of stress range zero and ultimate strength. The diagram is presented here in the hope that other experimenters will come forward with data to prove or disprove the hypothesis upon which it is based. Since we have no reliable means for determining stress and since fatigue tests on laboratory specimens cannot be used for evaluating the strength of machine parts, we e no recourse but to continue fatigue tests on machine parts in our industrial laboratories. There is, however, much that we can do to improve our technique in setting up the conditions of tests and in interpreting the test data.

Tests Based on Service Experience

The methods now used for coordinating laboratory tests with service experience are too haphazard to be completely reliable. Service failures must, obviously, be infrequent and when true fatigue failure does occur, it is the result of harder-than-usual service combined with a specimen lying on the lower fringe of the fatigue scatter band. Since failures must be infrequent, it is highly important that failed parts be examined by competent observers in order that the true cause of the trouble may be determined. Clear evidence of fatigue failure does not prove that the failed part was primarily responsible. A bolt may fatigue because it was not properly tightened during assembly; a gear may fatigue due to improper support or to a failed bearing; a crankshaft may fatigue due to inadequate or mal-adjusted vibration dampers; and so on without end. It sometimes happens, therefore, that immediate corrections are made to the wrong part and recognition of the true trouble may be greatly delayed.

Laboratory tests must not only duplicate service failure as to location of fracture, but must, in some cases, produce failure in approximately the same number of stress cycles if accurate life comparisons are to be made. This requires that we distinguish between normal operation stress and the relatively infrequent overloads that caused the failure.

For instance, rear axle automobile gears are, at worst, stressed at low-gear torque one cycle out of every thousand (according to Almen and Boegehold in *Proceedings*, American Society for Testing Materials, Vol. 35, Part 2, 1935, page 99). The lifetime requirement of such gears, therefore, is 100,000 cycles at low-gear torque, equal to about 30 miles of travel. Due to the scatter of test points this is nearly 250,000 cycles measured on the average fatigue curve.

Automobile chassis springs normally operate through a small stress range, but they must be designed to withstand a total of high stress cycles equal to the number of bumps that will be experienced by the hardest driver on the worst road. This is a relatively small number of stress cycles but the problem is aggravated by the fact that such springs are subjected to severe corrosion and to surface damage by stones. Hence experience requires an average lifetime of 100,000 cycles at maximum amplitude.

Clutch springs in town driving are deflected approximately 500,000 times during the lifetime of an automobile, but not always at the maximum amplitude. Therefore, a life of 500,000 cycles at full amplitude is a minimum requirement.

Fatigue data from machine parts can profitably be studied for the purpose of constructing empirical formulas by which load capacity eventually may be accurately calculated.

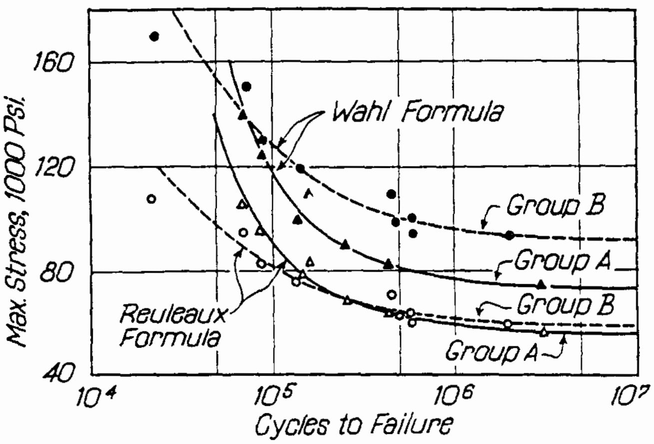


Fig. 15 — Edgerton's Tests on Helical Springs, Plotted on Semi-Log Coordinates. Group A and B are identical, except in diameter of helix. Stresses calculated by two formulas, and Reuleaux's seems to be more reliable as it gives fairly concordant results at long life

Comparison of Stress Formulas

An interesting comparison of the relative accuracy of two formulas for calculating stress in coil springs was made by C. T. Edgerton in "Stress in Helical Compression Springs, Present Status of the Problem" (*Transactions*, American Society of Mechanical Engineers, Vol. 61, 1939, page 643). He plotted semi-log S-N curves for two groups of springs that were identical in every respect except that they differed in the ratio of spring diameter to wire diameter, the spring indexes being respectively 3 and 5. The stresses for both groups of springs were calculated by the method of Reuleaux and by the Wahl method with the results shown in Fig. 15. From this chart Edgerton concluded that the Reuleaux method was superior because the curves for both springs gave approximately the same endurance limit whereas the endurance limit for the two springs calculated by the Wahl method varied by 20%.

When these data are plotted on logarithmic coordinates, as is shown in Fig. 16, and analyzed by the hypothesis previously stated, we cannot escape the opposite conclusion. Note that the curves plotted to the Reuleaux formula intersect at 300,000 or 400,000 stress cycles, an impossible condition by the proposed theory because the materials were identical. The Wahl formula curves intersect in the region of high stress as is required by the theory. The difference in slope of the two Wahl curves is interpreted as differences in stress concentration or in stress range for these two groups of springs. The fact that these data show greater stress concentration or greater stress range for the springs having the greater index requires explanation, but the data at hand are insufficient to permit a more complete analysis.

The numerical value of stress in these springs as shown in the chart is not now important for reasons that have been discussed, nor should it be assumed that the Wahl formula will suffice when adequate data have been studied.

These springs are introduced here to show that the accuracy of practical stress formulas for any machine element can be analyzed no matter how complex the stress pattern if given the necessary fatigue data. This has already been done for a number of machine parts and sufficient fatigue tests have been made on other parts to construct significant stress formulas if they can be assembled for study.

It is realized that the discussions given the several subjects introduced in this paper are inadequate and incomplete. They are inadequate because of limitations of space and they are incomplete because of meagerness of data. It is hoped, however, that enough has been said to indicate some of the shortcomings of our present procedures and to stimulate action whereby we may improve the efficiency of both men and materials.

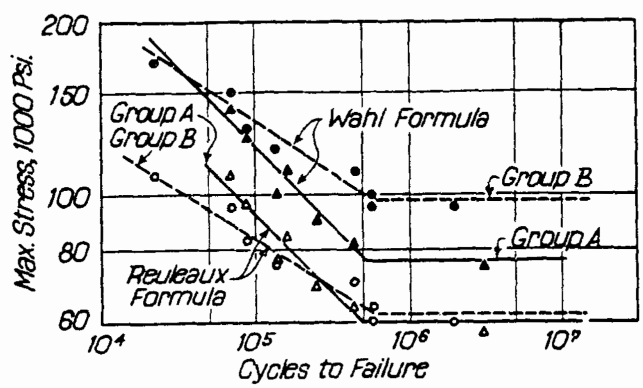


Fig. 16 — Same Tests Plotted on Log-Log Coordinates Indicate That Wahl's Formula Gives Better Agreement With Logic of Circumstances Existing at Heavy Stresses and Short Life

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