Fatigue Damage Measured by Deflections of Rotating Beam Specimens

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ABSTRACT

An apparatus was constructed to continuously record throughout the finite-life of a rotating beam fatigue specimen (1) the mid-span deflection of the specimen and (2) the torque required to keep the specimen rotating at constant speed. The deflection is a measure of the strain in the material and the torque is a measure of the internal energy loss due to plastic distortion. The significance and some results of the tests are shown.

The construction and calibration of the apparatus is described in detail.

INTRODUCTION

Fatigue specimens stressed above the linear elastic range behave elastically for a part of their finite-life. During the early stages of repeated loading the dynamic deflection of a specimen is not plastic. Depending upon the stress level, a number of cycles of stress must be applied before the deflection under dynamic loading becomes equal to the plastic deflection found under static loading.

This paper describes the apparatus constructed to measure and continuously record throughout the finite-life of a rotating-beam fatigue specimen, (1) the mid-span deflection and (2) the torque required to keep the specimen rotating at constant speed. The deflection is related to the dynamic stress-strain characteristics and the torque is a measure of energy loss within the material.

THEORY

Torque Required to Turn a Specimen

Fig. 1 shows the tensile and compressive stress-strain relations for Ti-75A titanium alloy. The stress-strain curve in compression is very nearly a mirror image of the curve in tension. The limit of linear-elastic behavior in tension is 52,000 psi, and in compression is 49,000 psi.

Consider a cross-section of a specimen stressed in pure bending within the elastic range. A linear distribution of stress is assumed. For every fiber which has a compressive stress there is a diametrically opposite fiber which has an equal tensile stress. The rates of change of stress in two opposite fibers are equal and opposite as the beam rotates in pure bending. At any time the total tensile load is equal to the total compressive load. With no internal energy losses, the loading cycle for each fiber will follow the linear-elastic line A-A' of Fig. 1. No torque, except that necessary to overcome friction, windage, etc., is needed to turn the specimen through a complete cycle of stress. The system is conservative. However, when the specimen begins to deflect plastically the system is no longer conservative. The energy associated with the increase in deflection due to plastic behavior is not stored in the system. This energy is dissipated as heat. If additional energy is not supplied to the system the specimen would slow down and eventually stop. To maintain the specimen at constant speed an
additional torque must be applied. Since this torque supplies energy to the system at the same rate that it is dissipated as heat to the surroundings, the torque is a measure of the internal energy loss within the material due to plastic distortion under repeated loading.

To measure the torque required to turn the specimen at constant speed, a flexible bar was placed between the motor and the specimen so that the torque required to turn the specimen must be transmitted through this bar. The bending strain produced in the flexible bar, a direct function of the torque required to turn the specimen, was continuously recorded during the tests.

Mid-Span Deflection of the Specimen.

Fig. 2 shows the rotating beam specimen used in this work. Roughly three-fourths of the total angle change occurs in the middle third of the specimen. This means that the deflection of a point on the bearing housing was nearly a linear function of the mid-span deflection. Measurements of mid-span deflection were made without touching the rotating specimen by calibrating the deflection of a point on the housing with that of the mid-span of the specimen.

CONSTRUCTION AND CALIBRATION OF APPARATUS

Figs. 3 to 6 are photographs of the apparatus. Fig. 3 is a general view showing both the torque and deflection recorders in position for testing. Fig. 4 shows the deflection indicator and recorder. Wires from an SR-4 gage (under friction tape) connect to the recorder in the upper right. Fig. 5 is a view of the loading apparatus. The resistor in the upper left is in the torque-measuring circuit for controlling the pen movement to full scale. Fig. 6 shows the torque meter in more detail. The copper contact disks rotate in a mercury bath in the plastic boxes. The wires from the rear of the boxes lead to the torque recorder on the far left.
Deflection Meter.

The deflection measuring apparatus is shown in the photograph of Fig. 4. A flexible cantilever beam was rigidly attached to the frame of the R. R. Moore fatigue machine to make contact with the bearing housing. An SR-4 electrical strain gage was attached to the cantilever beam. The strain at the SR-4 gage is a direct function of the deflection of the beam. The electrical bridge and recording circuit of a Sanborn Model 140 recorder were used to measure the deflection of a point on the bearing housing.

Calibration for Deflection.

A specimen of the standard dimensions as shown in Fig. 2 was placed in the apparatus and statically loaded. A dial gage was set up to record the deflection at the mid-span, while the cantilever beam deflection gage recorded the deflection of a point on the bearing housing.

Fig. 8 shows the relation between the dial gage and deflection gage readings. One deflection recorder division represents a mid-span deflection of 0.0055 in. Fig. 9 shows the mid-span deflection under static load as a function of stress. The mid-span deflection is 0.00141 in. per pound load or 0.00186 in. per 1000 psi stress at the extreme mid-span fiber in pure bending. The calculated deflection is 0.00145 in. per pound load.

Delay in Plastic Deformation During Calibration for Deflection.

The dial measuring the mid-span deflection responded quickly to the load on the specimen, and for loads within the linear-elastic range the dial indicator did not "creep". However, for loads above the linear-elastic range, that is for stresses above 50,000 psi, the indicator continued to creep after its initial movement. The initial dial movement, upon applying a load rapidly, but not by impact, was nearly that ex-
pected from an extension of the linear-elastic portion of the stress-strain curve. The data plotted in Fig. 9 were obtained from tests on individual specimens for each stress level. The stress in each specimen was increased from zero to a maximum with one application of the load.

Because the delay in plastic deformation is significant in this study, a load applying apparatus was set up to insure no static destruction of the fatigue specimen prior to a fatigue test. Fig. 5 shows the loading arrangement. The load was applied gradually during 25 revolutions of the specimen, but this rate of applying load could be altered.

Torque Meter.

Fig. 6 shows a photograph of the torque meter and Fig. 7 is an assembly sketch of this measuring device. Two shafts approximately 12 in. long were supported on self-aligning bearings. One shaft was attached to the housing of an R. R. Moore rotating beam fatigue machine by a coupling. The other shaft was similarly attached to a motor and variable speed transmission combination with a speed range from zero to 1100 rpm. The shaft attached to the housing has a rigid pin bar attached to its free end, while the shaft attached to the motor transmission has a flexible bar attached at its free end. The pin from the pin bar engages the flexible bar completing the transmission from the motor to the bearing housing. The strain produced in the flexible bar is a direct function of the torque required to turn the specimen. SR-4 electrical strain gages were attached to the flexible bar to record the strain. The leads from the SR-4 gages were brought to two copper disks, each of which ran in a bath of mercury. Leads from the mercury boxes ran to terminals of an SR-4 strain analyzer to complete the usual electrical bridge circuit. The strain analyzer was altered, however, and the unbalance of current caused by strains in the flexible bar
FATIGUE DAMAGE MEASURED BY DEFLECTIONS

was recorded on a Leeds and Northrup "Speedomax" continuous recorder.

Calibration for Torque.

Fig. 10 is a photograph of the setup for calibrating the torque meter. The shaft attached to the R. R. Moore fatigue machine was locked and the flexible bar subjected to known torques. A variable resistor, placed between the unbalanced current output of the SR-4 strain analyzer and the torque recorder, controls the sweep of the pen. Fig. 11 shows the relation between the movement of the pen and the applied torque for different resistor settings. These plots show the linearity of the entire torque measuring system. Based upon this linearity, a weight which applied a torque of 3.0 in.-lb was used to calibrate the apparatus before each specimen was tested.

The total energy was obtained from the torque record in the following manner: A torque of 3.0 in.-lb caused a lateral pen movement of \( x \) divisions. Then one division represents a torque of \( 3.0/x \) in.-lb. The applied torque acts through \( 2\pi \) radians for each revolution of the specimen. The total energy expended in one revolution is equal to the torque times the angular movement, or \( 6\pi x \) in.-lb. The recorder chart has a constant linear speed of two divisions per minute. The speed of the specimen is \( c \) revolutions per minute. Hence, the total number of revolutions of the specimen per chart division is \( c/2 \), and the total energy per chart square is \( 3\pi c/x \) in.-lb. Fig. 12 gives the total energy per chart-square as a function of the lateral pen movement \( x \) for testing speeds of 200, 400 and 900 rpm. Before each test the 3.0 in.-lb torque was applied and Fig. 12 was used to determine the exact energy per chart-square for that test.

It was not verified under dynamic tests that \( x \) divisions lateral pen movement represented the same torque as it did under static tests. It seems reasonable to assume that the torque is the same for dynamic or static load con-
ditions since the torque applied to the flexible bar increases slowly under the dynamic load.

Friction, Windage, Bearing Pressure Losses.

Energy losses from friction, windage, and bearing pressure are of no consequence provided they are constant and are small in comparison to the energy loss which represents a change in material structure. The following observations were made at each testing speed to check the energy losses:

(1) A specimen of large diameter was placed in the fatigue machine with no load. The movement of the recorder pen when the motor was turned on was one-half to one chart division, depending on the speed of the motor. (Full chart scale is ten divisions). This torque was attributed to bearing friction and windage losses. Loads which did not produce stresses above the linear-elastic range of the material were applied. These loads were much larger than those which would be applied to a specimen of regular size. There was no additional recorder pen movement upon application of these loads, nor did any movement take place after 30 minutes of operation. This showed that there was no change in bearing friction due to added load, and that there was no change in friction due to heating of oil in the bearings.

(2) A regular sized specimen of Ti-150A titanium alloy was placed in the apparatus with no load. The Ti-150A alloy has the same modulus of elasticity as the Ti-75A alloy used in this study. However, Ti-150A has a linear-elastic range of 92,000 psi while Ti-75A has a range of only 52,000 psi\[1\]. Thus loads could be applied to the Ti-150A which allowed large elastic distortions. The movement of the recorder pen when the motor was turned on was identical to that observed under (1) above. Loads were grad-

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* Superscripts in brackets pertain to references listed at the end of the paper.
ually applied. No additional movement of the pen was observed under loads which produced stresses less than 85,000 psi. Above 85,000 psi there was some small movement (about 0.3 chart divisions) which was attributed to minute changes in the internal structure of the material. This procedure took at least 30 minutes. From this observation it was concluded that the bearing friction did not change with load even for large angle change.

(3) The procedure in (2) was repeated with a standard Ti-75A specimen. Additional pen movement was noted when loads were applied which produced stresses of 30,000 to 50,000 psi. This movement, about 0.1 chart division, was attributed to minute changes in the internal structure of the material and was considered negligible.
FIG. 9. MID-SPAN DEFLECTION UNDER STATIC LOAD AS A FUNCTION OF STRESS—SPECIMENS HELD IN REGULAR POSITION FOR FATIGUE LOADING.

FIG. 10. SETUP FOR CALIBRATING TORQUE METER.
TESTS CONDUCTED

The apparatus described was used to conduct a study on the mechanism of fatigue damage. The results of this study are fully described in a report[2] to the sponsoring agency. Background fatigue tests were conducted at 30, 200, 400, 900, 1800 and 10,000 rpm with and without a coolant bath. Torque and deflection studies were made at 200, 400 and 900 rpm. Results show that a rotating beam specimen acts elastically for a number of cycles of stress above the linear-elastic range of the material. This was established by observing the mid-span deflection. The initial deflection is consistent with that predicted by an extension of the

![Graph showing torque calibration curves for various resistor settings.](image)

FIG. 11. TORQUE CALIBRATION CURVES FOR VARIOUS RESISTOR SETTINGS.

![Graph showing curves of total energy per recorder chart square at different testing speeds for a 3 in-lb applied torque.](image)

FIG. 12. CURVES OF TOTAL ENERGY PER RECORDER CHART SQUARE AT DIFFERENT TESTING SPEEDS FOR A 3 IN-LB APPLIED TORQUE.
linear-elastic portion of the stress-deflection curve. The plastic deflection as found under static loading does not occur immediately but rather the material exhibits a temporary stiffness with a consequent delay in the development of the static-plastic deflection. A number of cycles of repeated load must be applied before the mid-span deflection becomes equal to the plastic deflection found under static load.

This number of cycles is influenced by the speed and conditions of testing, the geometry of the specimen, and the structure of the material.

Based on these observations, it was postulated that damage from cyclic loading starts only when the deformation of a specimen becomes equal to the plastic deformation found when the load is applied statically.

**FIG. 13. CONTINUOUS TORQUE RECORD AND CONTINUOUS DEFLECTION RECORD FOR STRESS OF 90,000 PSI - Ti-75A TITANIUM ALLOY TESTED AT 400 RPM WITHOUT COOLANT.**
Fig. 13 shows a reproduction of a typical recorder chart of mid-span deflection and torque. The specimen was stressed to a maximum of 90,000 psi and tested at 400 rpm without a coolant. The initial deflection was 0.168 inches, consistent with that predicted from an extension of the linear-elastic portion of the static stress-deflection curve. The deflection did not reach the static-plastic value of 0.190 inches until 200 cycles of stress had been run. At this point, both the deflection and torque began to increase rapidly. The specimen fractured after 620 cycles.

From this test, the threshold of the damage region at 90,000 psi was set at 200 cycles. Tests of over-stress and under-stress to and beyond this number of cycles were conducted to determine the change in finite-life and hence the damage produced. The tests of over-stress and under-stress showed that there is no effect on the finite-life of a specimen when it is over-stressed or under-stressed at various stress levels for a number of cycles less than that required to produce static-plastic deflection (200 cycles in the case of 90,000 psi). However, the finite-life of a specimen is altered if it is overstressed or understressed a number of cycles beyond that required to produce the static-plastic deflection. These results established the transition from pseudo-elastic to plastic behavior as the threshold of severe damage.

SUMMARY

An apparatus constructed to measure and continuously record the mid-span deflection and the energy dissipated during cyclic loading of rotating beam fatigue specimens tested at various speeds is described.

Tests show that the material tested, Ti-75A titanium alloy, exhibits a pseudo-elastic behavior during early stages of repeated loading at stresses well above the usual elastic limit.

For the first portion of the finite-life of a specimen the deflection increases slowly with repeated load applications. Once the mid-span deflection becomes equal to the plastic deflection as found under static load, it increases rapidly and failure ensues. The transition from pseudo-elastic to plastic behavior has been established as the threshold of severe damage. Tests show that there is no measurable effect on the finite-life of a specimen when it is over-stressed or under-stressed at various stress levels for a number of cycles less than that required to produce static-plastic deflection. The finite-life of a specimen is changed if it is overstressed or understressed a number of cycles beyond the threshold of severe damage.

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REFERENCES
