AN EXPERIMENTAL INVESTIGATION OF THE
OBLIQUE IMPACT OF SPHERES UPON
SIMPLY SUPPORTED STEEL BEAMS*

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ABSTRACT

A comprehensive program of study concerning the impact of spheres upon simply supported beams at oblique angles of incidence has been initiated. The initial impact velocity of 150 ft/sec for a 1 in. diameter steel ball is such as to generate permanent indentation at the contact point. The present paper describes the equipment and instrumentation employed in the investigation as well as the results obtained for a typical test run. The experimental data permit the construction of the complete stress and deflection history of the beam and the trajectory of the ball, including the contour and topography of the groove created at the impact point. Quantitative information concerning the propagation of flexural waves, the excitation of mode frequencies and the amplitude of the displacements and stresses produced in the beam are derived from test results. The analysis of the data is based upon a resolution of the resultant forces into components normal and tangential to the beam.

The compilation of results from one hundred thirty test runs and correlation of this information with theoretical considerations will be reported subsequently.

INTRODUCTION

The central normal impact of spheres upon simply supported beams was first rigorously analyzed by Timoshenko [1]** who combined the equations of forced beam vibration with the theory of elastic contact stresses developed by Hertz [2]. This exact solution was later supplemented by various approximations due to Lemaitre [3], Zener and Feshbach [4], and Lee [5] who attempted to reduce the labor of the numerical calculation by imposing certain restrictions on the impulse exerted against the beam. Eringen [6] has generalized the formulation of this problem for various types of end supports, while Hoppmann [7,8] has applied similar techniques to derive expressions for the impact on elastically supported and multispans beams.

Experimental measurements of strain and deflection histories, contact times and the motion of the striking body have been confined to central normal impact against simply supported beams. In almost all cases, the relative impact velocity has been limited to magnitudes below 15 ft/sec. Such investigations have been pursued for various beam materials and cross sections by Mason [9], Dohrenwend, Drucker and Moore [10], Tusi and Nisida [11], Duwez, Martens and Clark [12], Hoppmann [13], Arnold [14], and Jelinek [15]. A comparison of these results with the Timoshenko theory or its various simplified versions generally indicated good correlation. Although Davies [16]

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** Superiors in brackets pertain to references listed at the end of the paper.
demonstrates the presence of permanent deformations at negligible impact velocities, this conclusion is not surprising. The energy required to produce the plastic indentation is small compared to the total impact energy and differs only slightly from that stipulated for a reversible elastic process. Consequently, an elastic description of both the beam vibration and the contact force appear to be adequate for these low impact velocities.

Experimental results at impact velocities of about 100 ft/sec have been compared with the Timoshenko theory and an analysis representing the impact on an infinite elasto-plastic beam at constant velocity [17]. Although the latter theory is primarily applicable to such phenomena as punching operations, the curves predicted by the two theories are quite similar both in shape and magnitude. A comparison with the data indicates that an analysis stipulating an elastic beam with a plastic contact force will probably yield a good description for materials with high elastic yield points, such as steel; but plastic beam deformations must be considered for impact at these velocities upon soft materials, such as copper. The appearance of a plastic hinge at the contact point is the upper limit of validity of elastic beam considerations.

The present investigation is concerned with the oblique impact of \( \frac{\text{1}}{\text{4}} \) in. diameter steel spheres upon simply supported beams of various cross-sections at angles of incidence ranging from normal to virtually glancing.

FIG. 1. TYPICAL BEAM ARRANGEMENT.
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The impact velocity of about 150 ft/sec is sufficiently high so as to produce significant plastic deformation at the contact point without the creation of a plastic hinge. This initial paper will be primarily concerned with the techniques of measurement, data analysis, and a description of the phenomenon as determined from test results.

APPARATUS AND INSTRUMENTATION

The ballistic arrangement consists of a horizontal air gun which shoots a ½ in. diameter ball bearing against the test beam. The end connections of the beam simulate simple supports and are mounted on a heavy bar which can pivot about a vertical axis. A heavy cast iron table supports the air gun and beam mounting trunnion.

A typical beam is shown in Fig. 1. In the present tests, the beam size was limited to a span of 30 in. and rectangular cross sections varying from ¼ in. x ½ in. to ⅓ in. x ⅓ in. The beam material was cold-rolled mild steel.

Two types of simple supports were utilized as shown in Fig. 2. The connection shown in (a) was subsequently replaced by that shown in (b) as this connection more nearly approaches a simple support.

A 16-mm Fastax movie camera operating at a known speed between 2000 and 6000 frames per second photographed the path of the ball and the vibrations of the beam in a 6 in. x 9 in. field. Film speed was determined by the flashing of a neon bulb which exposed the edge of the film 120 times per second.

From four to six strain gages were located on the front and rear surfaces of each test beam. In the run described in detail in this paper, No. 29, five of the six gages were placed in a longitudinal direction at distances of 0, 6 and 12 inches from the center. A gage was also located at the center of the beam in a transverse direction, while the 6 in. position was duplicated at the front surface and the opposite end of the beam.

A Hathaway SC-16A six-tube drum-type recording oscillograph with direct-coupled preamplifiers recorded the strain on 35-mm film mounted on a 5.6 in. diameter drum. This drum was driven by a 5000 rpm Veridrive unit producing a film speed of approximately 1900 in/sec as determined by a 10 kc timing wave.

The initial ball velocity was measured by the time interval between two pulses generated by the electromagnetic effect of the traverse of the projectile over two earphones. The rebound velocity of the ball was determined by the time elapsed between the piercing of two paper membranes on which a silver grid was painted. The potential drop due to circuit interruption provided the signals for this measurement. Rebound angles could be ascertained from the location of the punctured holes.

Groove impressions were determined by macrophotographing the contour with a magnification of 33. A wax impression of the groove was sectioned for a plot of the profile.

EXPERIMENTAL PROCEDURE

The beam was set at the desired angle of obliquity with the trajectory and the gun was
aimed within \( \frac{1}{4} \) in. of the beam center in such a manner that overlapping of indentations was avoided. After balancing and electrically calibrating all channels, the test was initiated by starting the Veridrive motor. Both Hathaway and Fastax films were allowed to reach full speed, and then the ball was fired.

Upon completion of the shot, the grid time interval for rebound velocity determination, the initial velocity measurement, the groove position on the beam, the angle of rebound, and any existing plastic bending in the beam were measured. The contour macrophotograph and wax impression of the groove were obtained subsequently.

In order to determine the relationship between bridge sensitivity and scope deflection, two types of calibrations were performed. One consisted of recording the scope deflections for a standard 3 millivolt amplifier input. The other involved the mechanical calibration of the entire strain measuring system. This was achieved by applying known deflections to the beam center and noting the response of the oscilloscope.

RESULTS

The strain gage record for Run 29 is presented in Fig. 3 to two different scales. The figure clearly shows the propagation of the initial impulse along the beam, the dispersion of the initial wave, and the "steady-state" strain propagation in the beam after reflection of the transient from the ends. Fig. 4 presents the original photograph of the plastic indentation.

The path of the ball can be seen in a sequence of frames taken with the Fastax camera, as shown in Fig. 5. Succeeding frames permit,
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with proper enlargement, the determination of the frequency of the fundamental mode of vibration of the beam.

Table I exhibits a diagram of the strain gage locations and a reproduction of each strain record with the salient features labeled. Values tabulated are the magnitudes and arrival times of calculated stresses, the velocity analysis, the beam vibration data obtained from the Fastax film, and groove dimensions. Certain additional computed results are also included: a coefficient of restitution, defined as the ratio of final to initial normal ball velocity; the normal and tangential loss of momentum of the ball; and an effective coefficient of friction, defined as the ratio of the tangential

FIG. 4. MACROPHOTOGRAPH OF GROOVE CONTOUR FOR RUN 29.

FIG. 5. FASTAX CAMERA RECORD OF THE IMPACT PROCESS.
TABLE I.
RESULTS FOR RUN 29.

VELOCITY ANALYSIS FROM FASTAX RECORD

<table>
<thead>
<tr>
<th>Angles</th>
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<tr>
<td>Initial Final Initial Final</td>
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| Coefficient of Restitution = \( \frac{V_{nf}}{V_{no}} \) |
| Tangential Velocity ft./sec. |
| Normal Velocity ft./sec. |
| Velocity Normal Velocity of Restitution = \( \frac{V_{nf}}{V_{no}} \) |
| -on- |
| -on- |

<table>
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<tr>
<th>MOMENTUM LOSS</th>
<th>EFFECTIVE COEFFICIENT OF FRICTION</th>
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<tr>
<td>( mv_1 ) lb-sec.</td>
<td>( \Delta mv ) ( (mv)_{avg.} )</td>
</tr>
<tr>
<td>0.0176</td>
<td>0.00103</td>
</tr>
<tr>
<td>0.00103</td>
<td>0.0406</td>
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STRAIN GAGE LOCATION
Arabic numerals refer to strain gage channels.

GROOVE ANALYSIS
BEAM VIBRATIONS

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<td>0.1002</td>
<td>0.1180</td>
<td>0.050</td>
<td>0.0058</td>
<td>0.00509</td>
<td>-0.099 -0.078 +0.058 0.058 -0.032 19.9 20.9 18.4</td>
</tr>
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<td>0.1002</td>
<td>0.1180</td>
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<td>0.00509</td>
<td>-0.099 -0.078 +0.058 0.058 -0.032 19.9 20.9 18.4</td>
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TRANSIENT STRAIN ANALYSIS: STRESSES AND TIMES

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<thead>
<tr>
<th>PEAK STRESSES, psi</th>
<th>TIMES, ms</th>
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<tr>
<td>Major Pulse (m)</td>
<td>ARRIVAL TIME</td>
</tr>
<tr>
<td>( n_1 ) ( n_2 ) ( n_3 ) ( n_4 )</td>
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\( a = \) Major Pulse Duration
\( b = \) Arrival Time of First Signal
\( c = \) Arrival Time of Peak of Predominant Wave Group
\( m = \) Major Pulse Stress
\( n_1 \), \( n_2 \), \( n_3 \), \( n_4 \) = Peak Precursor Stress
\( + = \) Tensile Stress
\( - = \) Compressive Stress

Ball Weight: 0.00057 slugs
Ball Diameter: \( \frac{1}{2} \) in.
Ball Material: Hardened Steel
Beam Material: Mild Steel

[176]
momentum loss to the average normal ball momentum. The penetration depth of the groove was calculated by using the measured width of the contour as the chord of a circle.

The time-history of the center deflection of the beam as determined from the Fastax record has been plotted in Fig. 6. It is evident from this diagram that only the fundamental mode frequency can be positively identified. Fig. 7 shows this same central deflection with the time scale expanded to correspond to the record of the central longitudinal gage (Channel 4) reproduced here.

**DISCUSSION**

The results presented in the preceding section provide a complete description of the processes involved in the phenomenon. The variables investigated may be subdivided into the following categories: (a) velocity history of the ball, (b) groove analysis, (c) determination of the lateral modes of the beam, and (d) stress analysis for the beam. These parameters are expected to exhibit significant changes upon variation of the angle of incidence and beam cross section. Certain de-
ductions concerning these trends, based upon the data of Run 29, have been substantiated by the results from other tests.

The velocity analysis of the ball trajectory permits the evaluation of the coefficient of restitution, the normal and tangential momentum loss, and an effective coefficient of friction. The coefficient of restitution as defined in this paper differs from that customarily employed by neglecting the velocity of the beam at the end of impact and thus provides a direct measure of the loss of normal momentum of the ball. Its magnitude will vary from some minimum value at normal incidence, which depends upon the initial ball momentum, the mass ratio, and the material properties of the colliding bodies, to a theoretical value of unity at glancing incidence. The loss of momentum of the ball provides the impulse which produces the beam vibrations and the groove. The ratio of normal to tangential momentum loss for this 70 degree shot clearly indicates the predominance of the normal impulse transmitted to the beam and, consequently, the amplitude of the longitudinal vibration is expected to be very small. The effective coefficient of friction varies from a peak value at glancing incidence to a zero value at normal incidence.

The ball would be expected to rotate as the result of the friction force applied tangentially to the sphere during the impact process. However, this rotation did not materially affect the linearity of the rebound trajectory, as determined both from the Fastax records and from observation of the piercing points of the grids.

The groove indentation will vary from a circular shape at normal incidence to line contact for glancing impact. Analysis of the groove dimensions permits a delineation of the trajectory of the ball during contact, and the tangential component of this path divided by the tangential velocity (which remains virtually constant) yields an approximation of the duration of contact. For this run, a value of 32 microseconds has been deduced in this manner. The number of these precursors increases with distance from the impact point prior to wave reflection from the ends, the earliest signal (in time) at any station being the precursor most recently created. The energy initially contained in the major pulse is transferred to these precursors. Upon arrival at any station of a signal reflected from the ends, the initial transient is terminated and the "steady-state" strain wave propagation is initiated. The phenomena in this domain are complicated by continued reflection and dispersion of the pulse, and consequently only predominant frequencies have been determined in this region.

A detailed analysis of the transients at the various gage stations, shown in Fig. 3, is presented in Table I. The salient wave characteristics tabulated include the amplitude, arrival time, and duration of the major pulse and the amplitude and arrival time of the precursors. Based upon the arrival of the earliest measurable precursor, the velocity of the
original bending wave is about 17,000 ft/sec, and the velocity of the reflected wave based on corresponding data is about 15,000 ft/sec. On the other hand, the average velocity of the original and reflected waves, based upon pulse durations, is computed at 10,000 ft/sec. The difference between these values is due to presence of both longitudinal and flexural wave components traveling, respectively, at these velocities; the predominance of the flexural component would be expected to primarily affect the pulse duration.

Additional evidence of the existence of a longitudinal wave is a small disturbance in the flat portion of the transient of the central longitudinal gage. This checks very closely the transit time for a "rod" wave, whose velocity is 16,800 ft/sec, to propagate from the center to the end and back. The phenomenon described is not an artifact, but has been observed on all records to date.

The highest stress attained throughout the process occurred at the center of the beam with a peak transient value of +21,800 psi. This stress is well within the elastic range even when the stress-strain curve is derived from static tests. The peak transient diminished with distance along the beam. The maximum stress during the "steady-state" strain propagation in the beam was also recorded by the central longitudinal gage and amounted to about 60 percent of the peak transient value. Gage 3 was positioned (see diagram, Table I) on the front face of the beam directly opposite Gage 5 for the purpose of determining symmetry with respect to the neutral axis. Gage 2 was placed symmetrically to Gage 5, but on the other end of the beam, to investigate longitudinal asymmetry of beam response. The strain records obtained from Gages 2 and 3 are virtually mirror images until shortly after the establishment of the free vibrations. This demonstrates that the bending stresses predominate over the longitudinal wave stresses. The record from Gage 5, shown inverted in Fig. 4, is considerably in error due to a severe unbalance in the push-pull stages of the oscilloscope amplifier.

A transverse gage, No. 1, was placed at the center of the beam to provide additional information concerning the character of the initial pulse. The gross appearance of this record approximates the mirror image of the central longitudinal gage to a smaller stress scale, which probably represents a Poisson effect. In addition, several cycles of an extremely high-frequency oscillation of 180,000 cps appear superimposed on the initial stages of the transient. This frequency is 10 percent below that of a "rod" wave and 22 percent below that of a dilatational wave traversing the thickness of the beam. Although the frequency is faithfully reproduced, the amplitude at this frequency is attenuated due to the upper cutoff of the amplifier. This observed frequency substantiates a time axis shift of 2 microseconds to account for the traverse of the beam thickness by the initial stress pulse.

The beam deflection is produced by the momentum transferred from the striking ball. The variation of the central deflection with time is a slowly decaying sine wave of the fundamental mode of vibration of the beam with higher harmonics superposed.

There is no lateral motion of the beam at any station until the arrival of the earliest precursor of the bending wave. Consequently, the fixity of the end supports has no effect on the lateral beam motion until the arrival of the earliest signal at the ends. By the same token, all gages at positions other than the ends of the beam are not affected by the type of end support until the arrival of the reflected lateral wave. This confirms a conclusion of Dohrenwend, Drucker and Moore [10] in experiments involving beams with much longer length-to-depth ratios.

A careful determination of the accuracy of the strain gage records indicates a maximum possible error in the stress magnitudes of ±5 percent. Velocity and time parameters could be ascertained to within ±2 percent, while the groove dimensions are accurate to within ±1 percent.

ACKNOWLEDGEMENT

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REFERENCES


