An Experimental Investigation of Residual Stress in Quenched Flat Plates

Paper constitutes a summary of investigation conducted by the authors and encompasses the experimental-apparatus design, the heating and quenching of test specimens, the recording of the time-temperature history of the plate during cooling, and the experimental measuring of the residual stresses which have been induced in the plate by the quenching

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ABSTRACT—The residual stresses induced in a quenched flat plate are measured experimentally. A flat plate approximating the theoretical infinite flat plate, after being heated to a uniform temperature, is quenched uniformly from both sides in order to cause a residual-stress distribution. The residual stresses are obtained by successively grinding the plate on one side and measuring the resulting strains on the opposite side. Experiments are performed on plates experiencing no phase transformation—316 stainless steel. Comparison of results is made with the predicted stress distributions from the analysis of Landau and Weiner, which is derived from the time-temperature history of the plate during its quench.

List of Symbols

E = Young's modulus, psi
u = depth of cut, in.
S, σ = stress, psi
ϵ = strain, in./in.
α = depth of plate under consideration, in.
ν = Poisson's ratio

Subscripts

a = axial
b = rear of plate
m = moment
t = upper surface of plate
α = depth of plate under consideration
X, Y = orthogonal directions in plane of plate
u = depth of cut

Introduction

The use of, and sometimes the disastrous effects of, residual stresses in many heat-treated machine parts promotes great interest in possible methods of predicting residual-stress distributions for any given heat treatment and thereby controlling residual stresses by choice of heat treatment. The bulk of the published literature on this subject deals primarily with the measurement of residual stresses by various techniques in a number of configurations. Lynch[1] describes in some detail many of the methods, including those of Sachs, Baldwin, Bauer and Heydn, Mesnager, Stablein, Davidenkov and Espsey. All of the aforementioned utilized destructive tests—either slicing, machining or drilling the specimen. A great deal of the work of Bauer and Heydn, Baldwin, Mesnager and Sachs has been on cylindrical shapes. The methods utilized by all of these investigators involves measured data in the form of strains as a basis for residual-stress determination.

For geometries in which the residual stress varies in one direction only, such as a flat plate infinite in length and width, Stablein, Davidenkov, Sachs and Baldwin measured strain as a variable, along with the amount of material removed from the specimen. The strain, in the case of a flat plate, can be measured directly by strain gages or by measuring deflection of the specimen as strips of material are removed. The latter method is employed by Baldwin[1] and by Treuting and Read.[2]

Rosenthal and Norton[3] have developed a method of residual-stress calculation which involves slicing the stressed specimen from one side and measuring the resulting strain directly with strain gages on the opposite side of the test piece. A modification of this method is utilized in this investigation.

The approach adopted in the present study was to introduce residual stresses in a mathematically tractable geometry under carefully controlled con-

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ditions and amenable to calculation, and then to determine and compare theory and experiment. This was considered to be the first step in the development of reliable methods for predicting and controlling residual stresses in engineering structures.

Experiments described herein have been conducted to determine the residual stresses induced in flat plates rapidly cooled from both sides and to compare these measured stresses with those calculated by the methods proposed by Landau and Weiner.[4] In calculating the residual stresses, the real temperature history of the cooling process is used. The geometry of flat plate, which lends itself to uncomplicated experimental procedures in measuring the residual-stress distributions, causes it to be one of the easiest to analyze with respect to transient temperatures during cooling. A material undergoing no phase transformation (AISI Type 316 stainless steel) and a material experiencing a phase transformation (steel with the alloying constituents being nominally 1-percent chromium, 1.25-percent molybdenum, and 0.25-percent vanadium) have been utilized experimentally, although the present paper deals only with the results of the stainless steel. This paper represents a summary of the investigation and encompasses the experimental-apparatus design, the heating and quenching of test specimens, the recording of the time – temperature history of the plate during cooling, and the experimental measuring of the residual stresses which have been induced in the plate by the aforementioned quenching.

Materials and Apparatus

Desirable mechanical properties for test materials selected for comparison with analytically tractable results (where there is no phase transformation) are: (1) a well-defined yield point, (2) absence of strain hardening, and (3) a yield point high enough that residual stresses can be produced and can be measured. The material chosen was AISI 316 stainless steel which represented the best compromise in these properties. The yield stress, while not constant over the temperature range to which the material was subjected, remains above 16,000 psi even at the maximum temperature employed, 800° F. Figure 1 shows the yield-stress variation with temperature for this material.

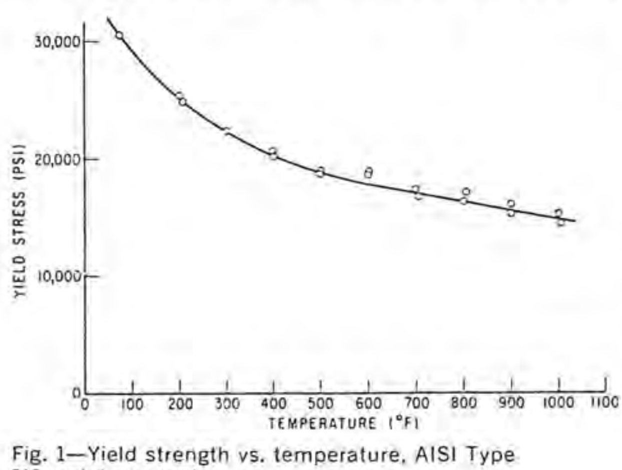

Fig. 1—Yield strength vs. temperature, AISI Type 316 stainless steel

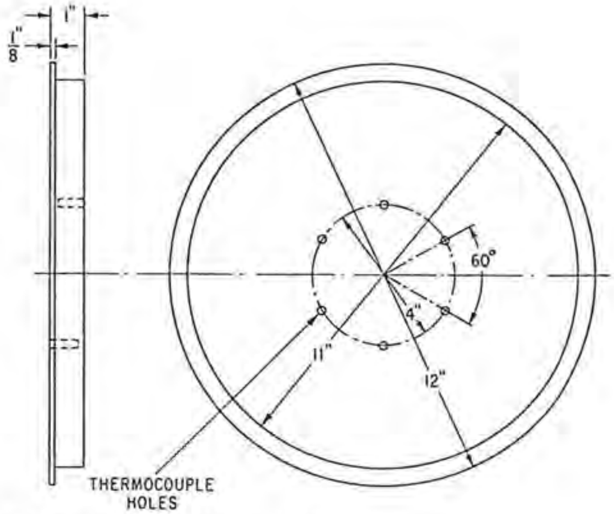

Fig. 2—Test-plate configuration

The test-specimen geometry approached that of a flat plate infinite in length and width, having a minimum thickness to width (length) ratio of one to eleven. To insure enough thickness to allow measuring of stress changes at different levels of depth by surface removal, an initial thickness of 1 in. was chosen. The resulting test plate, shown in Fig. 2, is a circular plate 12 in. in diameter and 1-in. thick, with a 1/8-in.-thick lip 1/2-in. wide on one surface.

Chromel-Alumel couples proved satisfactory for measuring transient temperatures during cooling. Flexible, glass-covered 10-mil wire worked very well for temperatures up to 1000° F. To mechanically protect the couples from water spray, the entire couple was placed in a Kovar tube 0.090-in. OD. The recording of temperatures inside the plate necessitated holes, large enough for the ceramic insulators, being drilled into the plate at discrete depths. A capacitance-discharge welder was used to weld the thermocouple bead to the bottom of the hole.

To record the temperatures, a Model 5-116 P414 Consolidated Engineering Corp. recording oscillograph was employed. This permitted the simultaneous recording of six thermocouples.

To heat and then cool the plate, a large furnace-quenching apparatus was built (Fig. 3). A supporting device for the test plate was placed in a 30- by 48-in. tray. To insure that the plate would be cooled from the faces only and not from the edges, a supporting ring fitting the plate snugly and enlarging the effective plate area to a 14-in. square was fixed vertically in the center of the tray. Directly above the plate, a rectangular furnace was suspended in a system which allowed it to be raised and lowered handily over the plate. The

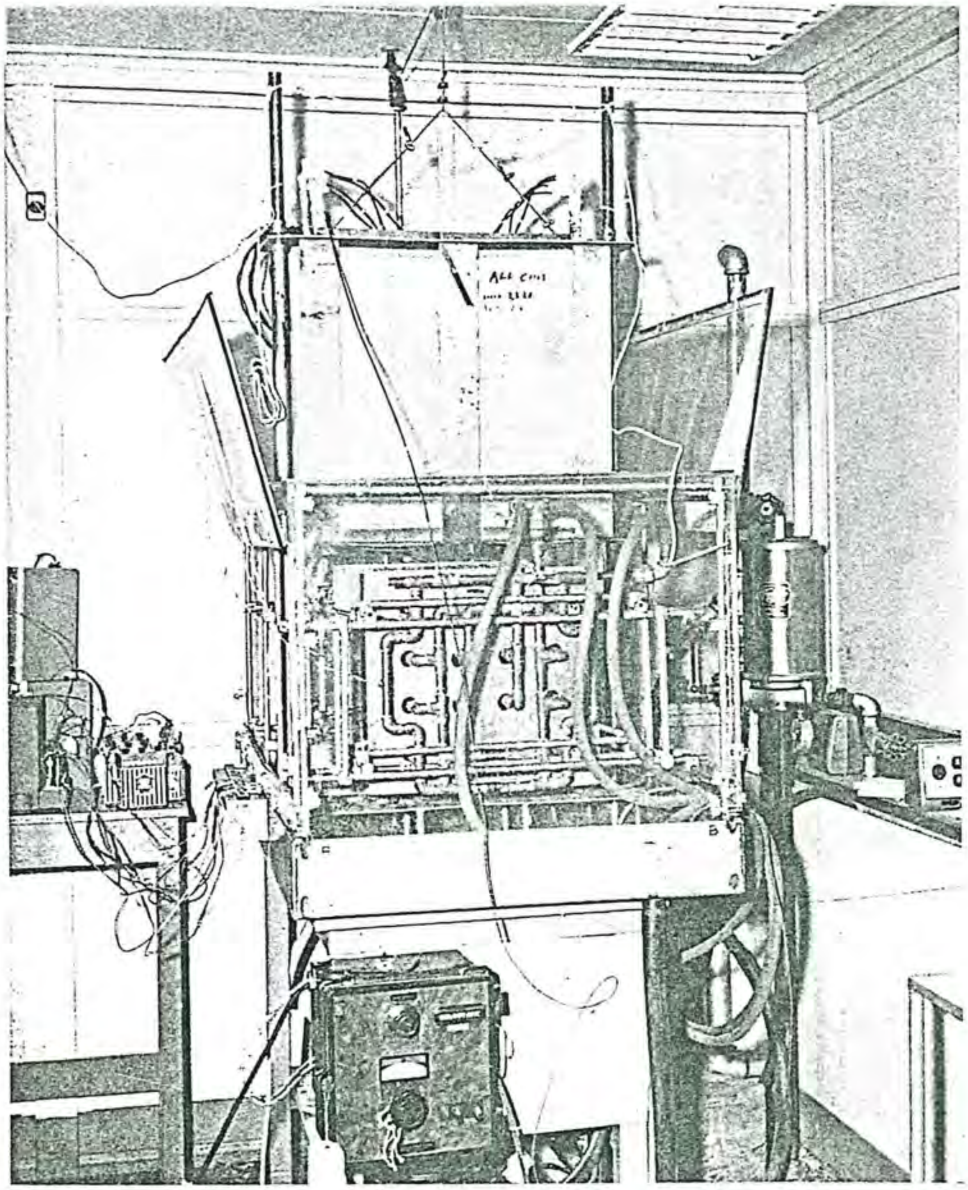

Fig. 3—Heating-quenching apparatus, showing furnace in raised position

furnace had four separate heating coils wrapped horizontally around a rectangular stainless-steel core. By experimentally adjusting the voltages across each coil, a uniform temperature of the plate could be obtained. The base on which the open bottom of the furnace rests was Transite, providing a good seal against heat loss. The temperature of the furnace is controlled by a Celect Ray controller, actuated by a Chromel-Alumel thermocouple inside the furnace.

To provide a uniform cooling rate on both sides of the plate, a battery of 16 nozzles fixed on a horizontally moveable bracket was placed on each side of the plate. Altering the nozzle-to-plate distance allowed variations in boundary conditions for the quench. The nozzles, fixed in a square pattern, were fed from three $\frac{1}{2}$-in. lines. The most satisfactory nozzles found were WSS 10 Delavan nozzles, solid-cone spray type. Over each bank of nozzles was built a covering plate fitted with a sliding panel which, when pulled away suddenly, released spray from each nozzle in a bank simultaneously. The covering plates were equipped with pull cords which allowed both banks of nozzles to be opened together. The bank of nozzles, along

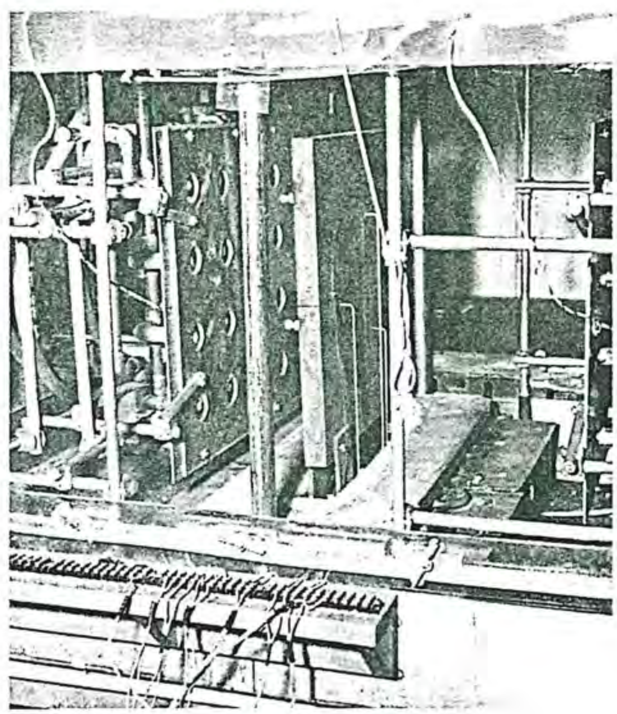

Fig. 4—Test plate in position in heating-quenching apparatus. (Note nozzle cover plates and thermocouples)

Fig. 5—Test plate and clamping fixture for grinding operation

with a plate in position ready to be heated, is shown in Fig. 4. The quenching apparatus, which delivered up to 35 gal of water per minute at 78-psi line pressure, could also be used as an air-quenching device.

As discussed below, the method of residual-stress measurement involved successive removal of surface layers in the quenched plate, measuring the strain for each step. The problem of removing metal from the plate to measure physically the residual stresses proved a major one. Machining and various chemical etching and electro-polishing techniques were first tried but were unsuccessful. A Blanchard grinder, with a table speed of 25 rpm and a grinding-head speed of 705 rpm, fitted with four LA248J2VC grinding segments (Robertson Manufacturing Corp.), provided a satisfactory method. The coolant used was an all-purpose wax-base liquid. To hold the test plate on the magnetic table of the grinder, a 3-in.-thick base plate was built. A clamping ring, bolted to the base plate, held the test specimen flat (Fig. 5). A recess in the base plate allowed the presence of strain gages on the rear of the test specimen. The strain gages proving reliable were Baldwin SR-4-AB-1 gages. Satisfactory insulating materials were Gasco Neoprene kit components. A Baldwin SR-4 strain indicator was used to record strains.

Theory

Since the test specimen is quenched equally from both sides, the resulting stress distribution is symmetrical with respect to the central plane of the plate. Ideally, it is only necessary to measure the stress in one half of the plate to corroborate results.

When the heated stainless-steel plate (which involves no phase transformation) is cooled rapidly from the outside, this outer portion tends to contract. Because the hotter inside of the plate is still thermally expanded, tensile stresses are set up in the outside portions and compressive stresses in the inside. To relieve these stresses, the outer surface flows plastically. As temperature equilibrium is reached, the inner portion now tends to be too small for the outside portions. Thus, residual tensile stresses are set up in the inside of the plate and compressive stresses in the outside portion. According to Weiner,[5] the possible shapes of residual-stress distributions resulting from such a quench are as shown in Fig. 6. The residual stresses shown in this graph are dependent on the severity of the quench.

In the case of a material with a phase transformation, the effects of heat liberation and volume expansion during transformation complicated the process such that it may be possible to find tensile stresses in the outside portions of the plate. The heat liberation during the transformation alters the transient-temperature distribution; the volume change further influences the stress distribution such that the final residual-stress distribution may be entirely different from the distribution evidenced in the simple case.

Landau and Weiner[4] have been able to predict the stress distribution in quenched flat plates made from materials not experiencing phase transformations. The predicted stress distribution is gained from a knowledge of the quenching parameter called the Biot Number, defined as the product of the surface heat-transfer coefficient and the half thickness of the plate divided by the thermal conductivity of the plate material. The simplest calculation for predicting residual-stress distribution would involve a quench where the boundary conductance is constant. However, any arbitrary quench can be divided into a series of time intervals, each with a constant Biot Number, if the time–temperature history of a portion of the plate is known throughout the quench.

The initial goal of the experiments described in this paper was to produce, by spraying water at both sides of the plate, a quench of a constant Biot Number. The minimum requirements of the experiments were to produce, first, identical boundary conditions on both sides of the plate during the quench, and secondly, to produce a uniformity of quench over the entire surface of the plate. With a time-temperature history of the metal at an arbitrary depth of the plate during such a quench, the residual-stress distribution could be calculated analytically since the value of the Biot Number can be found as a function of time, and the residual stresses can be calculated from knowledge of the Biot Number. Because the Biot Number calculations are very sensitive to temperature gradients, the most precise results can be gained from temperature-time data recorded near the surface of the plate. However, surface-temperature readings are subject to error. For this reason, most of the transient temperatures measured in this investigation were at a depth of 0.1 in. in a 1.0-in.-thick plate.

The experimental measurement of the residual-stress distribution involved destructive testing, that of removing metal incrementally from one side of the plate while recording the changes in strain on the opposite side of the plate. An analysis for such an experiment has been done by Rosenthal and Norton.[3] As the stressed material is removed from the plate, an equivalent moment and force are removed from the plate, causing the plate, when it is not restrained by any clamping device, to deflect and develop a curvature. From a knowledge of the strain accompanying this removal of material from the opposite side, the original residual-stress distribution is calculated (see Appendix). The strain measurements recorded include the strain reading on the rear of the plate after the plate had been sliced and allowed to lie unrestrained in an unclamped position, and the strain measurement, from the same strain gages, taken after the plate was reclamped and readied for the next cutting operation. The difference between these two strain readings and the exact depth at which these readings were taken provide the raw data from which the residual stress is calculated. It is assumed that the clamping action, once the plate has been allowed to lie unrestrained, will replace the moment necessary to remove all curvature from the plate, but will not replace the original tensile or compressive residual force relieved when the last piece of material was removed.

In the process of grinding away material from the face of the plate, residual stresses of some magnitude are induced in the plate near the surface. When a layer of material is removed from the plate, not only are the residual stresses associated with that layer relieved from the plate, but new stresses are put into the plate, thus complicating the measurement of the original residual-stress distribution. Consideration was given, in the analysis of data, to these stresses.

It was noted that removal of the initial layer of

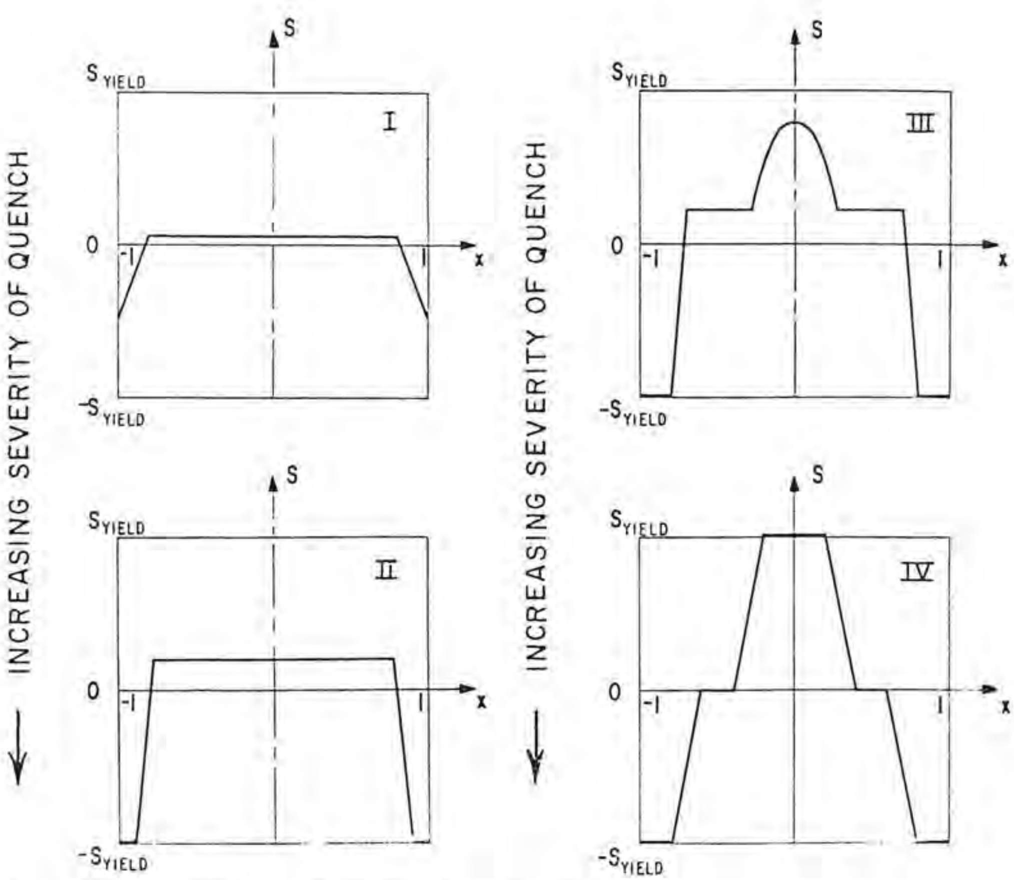

Fig. 6—Possible analytical residual-stress distributions

material from each plate, no matter how thin, produced measurable strains, resulting in essentially initial non-zero strain readings for zero depths of cut.

Further removal of material resulted in a finite-slope strain–depth curve. Several such curves were compared in order to determine what experimentally measured value of strain should be subtracted from each strain–depth-of-cut trace so that the resultant curves would possess both an initial finite slope and a zero strain value. Also, since the equation utilized to calculate stress distribution from strain data contains only the slope of the strain–depth curve for the stress at the surface of the plate, and since this stress could not exceed the yield stress of the material, a maximum slope of the strain-depth curve could be ascertained.

It was assumed that the process of grinding produces the same stress-distribution pattern at the ground surface of the plate no matter what the remaining thickness of the plate may be. This being the case, it is possible to calculate what shape the measured strain curve should take as a result of the grinding operation. Such a curve would be:

$$\epsilon_0 = K \div (1 - u)$$

u being the depth at which the grinding is done. The value of K in this equation would represent the correction factor previously described due to the initial grinding operation, the test value of which was deemed to be 38 μin./in.

Thus, the final graphs of strain vs. depth from which the residual-stress calculations were made had, as an ordinate, strain measured on the rear of the plate as the plate lay in an unrestrained condition minus strain measured on the rear when the plate was in the clamped position minus the value $38/(1 - u)$ μin./in.

Test Procedure

Six holes, each 0.070 in. in diameter, were drilled into the circular test plate. Five of the holes were drilled to a 0.900 ± 0.003-in. depth. The holes, three on each side of the plate, were spaced 60 deg apart and on a 3.5-in. radius from the center of the plate. The holes were drilled with flat bottoms to insure particular accuracy as to depth.

The plate was then stress relieved, at 500° C for 8 hr, in a dry-hydrogen atmosphere and then cooled at approximately 40° C/hr. After thorough cleaning of the surface of the plate and of the drilled holes, the plate was mounted into the holding device in the quenching apparatus. The Chromel-Alumel thermocouples, fixed with a junction approximately the size of the wires themselves, were placed in position. Since the bead was guided by its insulation tube to the center of the hole, and since the hole had a flat bottom, this insured that the bead was at the position desired. The capacitance-discharge weld provided a strong bond between plate and thermocouple. A minimum time lag was recorded by the thermocouple due to the small wire size and the good bond between the junction and the plate.

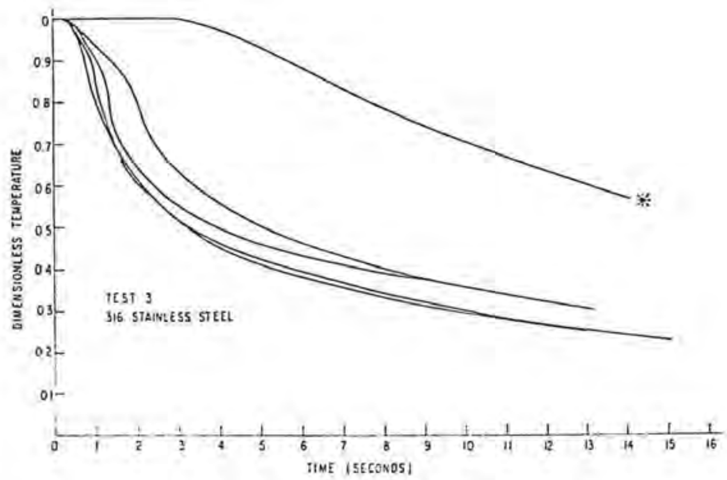

Fig. 7—Typical dimensionless temperature vs. time traces

The six temperature-measuring thermocouples were supplemented by an extra thermocouple welded to each side of the plate, which defined the moment when the quench began. No attempt was made to measure the surface temperature of the plate during the quench. When the thermocouples had been connected to the galvanometers in the recording oscillograph, the furnace was lowered over the plate.

When the plate was uniformly at the correct temperature, it was quenched in the following manner. The oscillograph was set to record. After the water or air spray was turned on, the furnace was lifted. Then the doors covering the nozzles were opened simultaneously. The plate was allowed to cool to room temperature under these conditions.

After removal from the supporting fixture, the quenched plate was lightly sandblasted on the surface in the region where the strain gages were to be affixed. Three SR-4-AB-1 strain gages were then cemented in radial directions with equal angles between them within a 4-in.-diam circle at the center of the rear of the plate, i.e., the side of the plate with the lip, and wired in series. Also, in the vicinity of the strain gages, a Chromel-Alumel thermocouple was firmly welded to the surface of the plate. The thermocouple attached to the plate makes it possible to measure the temperature of the plate at the time the strain readings are taken.

A smaller plate of the same material, affixed with a similar pattern of the same type strain gages and a thermocouple, served as a base for the compensating gages in the strain-gage circuit.

A recording of the strain measurement was taken at this time with the plate in an unconstrained position. The plate was then fixed into the grinding attachment. The strain was measured with the plate constrained. If the plate was exactly flat, the

TABLE 1—TEST CONDITIONS

Test number	Quenching medium	Initial plate temp. (° F)	Nozzle to plate distance (in.)
1	Water	800	$4^3/_4$
2	Water	800	7
3	Water	800	$4^3/_4$
4	Water	800	$7^3/_4$
5	Water	800	$9^{15}/_{16}$*

* Corner nozzles on each bank of nozzle plugged.

difference between the two strain readings should be negligible. Whatever the difference was, it became a correction factor which must be taken into residual-stress calculations. The test plate was now ground on the Blanchard grinder with a downfeed of 4 mils/min, a grinding-head speed of 705 rpm, and a table speed of 25 rpm. During the grinding, the plate was constantly under a stream of coolant. For the first 0.050 in., grinding was accomplished in increments of 3 or 4 mils at a time. From this depth up to approximately 0.10 in., the grinding increments were approximately 0.01 in. Beyond this depth, the grinding increments were approximately 0.02 in. Between each grinding operation, an accurate depth reading was made, recording the amount of material removed by all previous grinding operations. The plate was allowed to rest in an unconstrained position until the temperature of both the plate and the compensating gages reached approximately room temperature. When the temperature difference between test plate and compensating plate was less than 2° F, a strain reading was taken of the plate in the unconstrained position. Immediately, the plate was reclamped flat to the grinding apparatus, and a strain reading taken. The plate was then reground. Similar strain readings, taken between each grinding operation, comprised the raw data gained from the experiment.

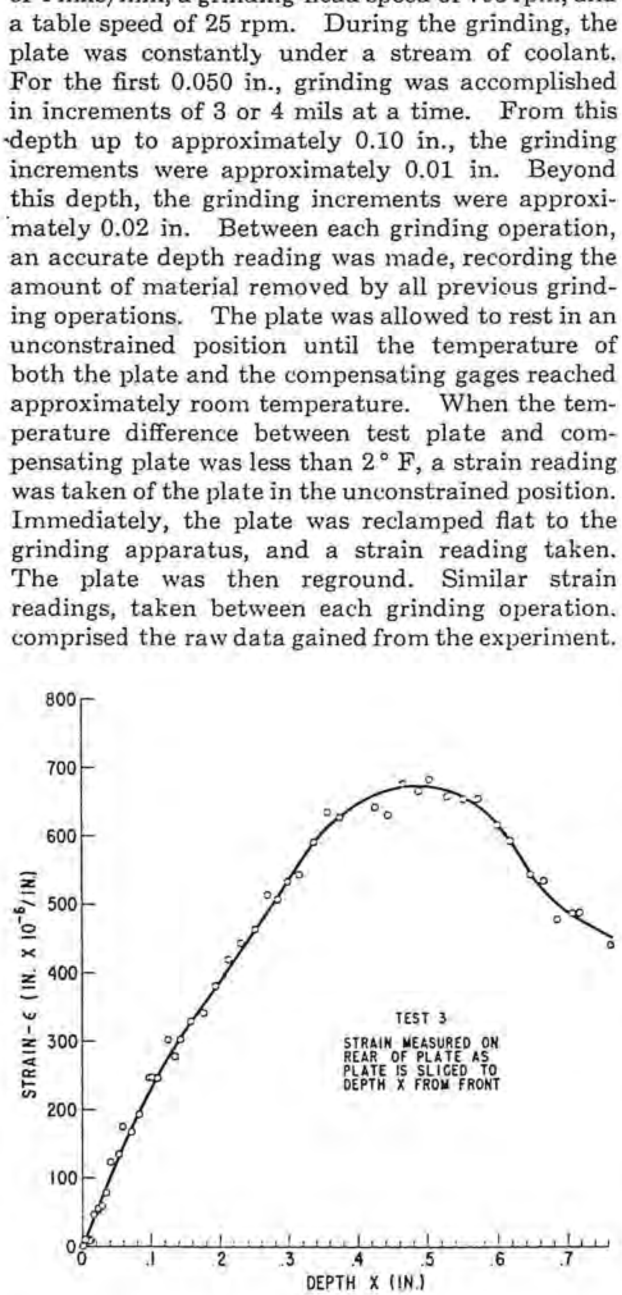

Fig. 8—Typical strain vs. depth-of-cut curve

The plate was ground until the remaining thickness was 0.22 in., which is the minimum thickness possible while preserving a lip on the plate itself and on the clamping device. Any further reduction in thickness would harm the test apparatus.

The data gained from the grinding experiment, strain change and depth of cut, were plotted, and the stress distribution for the plate at hand calculated.

Results

Listed in Table 1 are the test conditions for each 316 stainless-steel plate quenched.

Shown in Fig. 7 is a typical set of dimensionless temperature vs. time curves for the plates during their quench periods (test number 3 in this case). The dimensionless temperature is defined as the difference between the measured temperature and the cooling-fluid temperature, divided by the difference between the initial plate temperature (800° F) and the cooling-fluid temperature (approximately 50° F). The temperature traces shown were recorded at a depth of 0.9 in., except that noted by an asterisk, which was imbedded to a depth of 0.5 in., i.e., to the center plane of the plate.

It will be noted that the temperature traces evidence a "bump" early in the cooling time. This change in slope of the curve represents the radical change in the boundary conductance as the protective layer of steam from film boiling is broken down. A variation among similarly recorded temperature curves can be seen and is due to a lack of uniformity of boundary conductance over the surface of the plate.

Figure 8 shows the resulting strain vs. depth of cut relationships for the same test. The curves drawn represent the best curve drawn through test points. The presence of some scatter of data points presented a major problem in analysis of data.

Figure 9 illustrates the residual-stress distribution for the plate as calculated from the above strain-depth-of-cut curves by eq (A-11) in the Appendix. In some cases (not shown), the stress rises positively and then decreases, before rising again at the axis of symmetry of the plate. Each curve displayed a lack of symmetry showing that there exists some error in the strain curve used for a basis of calculations. The residual-stress distribution curve is also handicapped by the inability to complete the curve to the rear surface of the plate.

Discussion of Results

The dimensionless temperature vs. time curves displayed some differences between temperature traces recorded at identical depths within the test plate. The difference between the results for a given quench is explained by the inherent error in experimental procedure. The temperature-measuring thermocouples, while all imbedded in

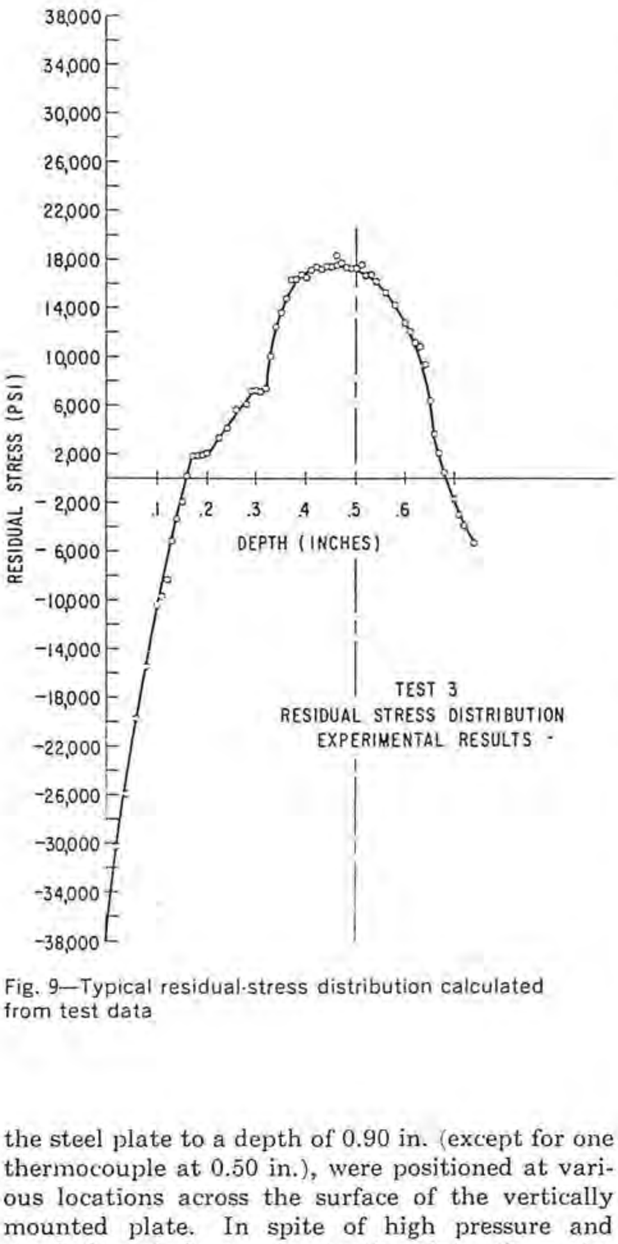

Fig. 9—Typical residual-stress distribution calculated from test data

the steel plate to a depth of 0.90 in. (except for one thermocouple at 0.50 in.), were positioned at various locations across the surface of the vertically mounted plate. In spite of high pressure and proximity of the nozzles to the plate, the water spray caused a water film to cover the plate. The thickness of the film increased near the bottom of the plate due to cascading of the water striking the plate near the top. The film tended to protect the plate from the impinging water, thus reducing the heat transfer from the plate on which the film was thickest. For this reason, the individual temperature–time curves chosen for analysis were those recorded in the vicinity of the horizontal diameter of the plate. Such a reading represents a mean cooling curve for the entire plate.

The value of the Biot Number for each test during the latter stages of the quench was found to be approximately six. In the early stages of each quench, the Biot Number, and thus the heat transfer, was somewhat less, varying from 0.5 to 1.5 for various quenches. The time of transition from low-quenching parameter to high-quenching parameter could be seen on the various dimensionless temperature–time curves as the obvious change of slope of the curves occurring as soon as 0.3 sec and as late as 3.5 sec (at about 1.3 sec in Test 3, Fig. 7).

By the analysis of Landau and Weiner, Zwicky[6] has shown that the residual-stress distributions in all the 316 stainless-steel specimens are very similar, showing an independence of the time at which the change in Biot Number occurs. The general shape of the analytical stress distribution resembles that of the third analytical residual-stress distribution shown in Fig. 6. The depth at which the stress changes from compressive to tensile is about 0.16 in. In all calculated stress distributions, the compressive surface stresses reached the yield stress, as evidenced by a region of constant residual compressive stress near the surfaces of the plate. Also evident in each analytical result was a region of constant tensile residual stress existing from about 0.21-in. depth to approximately 0.31 in. from the plate surface. Analytically, there existed a tensile residual stress of about two-thirds yield stress.

A study of the experimental residual stress for the 316 stainless-steel quenches, as calculated from strain data from the plate-grinding process, showed that all displayed the expected compressive residual-stress distribution at the surface. The magnitude of the compressive surface stress varied from the yield stress (approximately 40,000 psi) to about 25,000 psi. The depth where the residual stress becomes tensile varied from less than 0.1 in. to almost 0.2 in. All residual-stress plots evidenced a leveling out of the stress in the tensile region, similar to the analytical curves. The peak tensile residual stress, occurring at the middle of the plate, varies from about 15,000 to about 20,000 psi, falling somewhat below that of the analytical comparable stress.

While the experimental residual-stress curves do not agree precisely with the analytical stress distributions as predicted by Landau and Weiner, the experimental results do show the predictions to be valid. The general shapes of experimental and analytical curves resemble each other. The points of inflection and the peak magnitudes are not unreasonably distant.

A lack of symmetry of the experimental curves about the center line of the plate is accredited to the increasing error in data recording as the plate is sliced to a thin remaining section. Data taken during this phase are affected more by localized grinding residual stresses arising from the small remaining depth in which to dissipate grinding heat. Also, the strain gages displayed more drift when recording the large strains in this region.

The transition from raw strain data to the final experimental residual-stress curves involves a certain amount of error. The strain data evidence some scatter. Thus, the strain curve used for calculation represents an approximation. The approximation affects the strain magnitude and strain integral portions of the stress eq (A-11) to a small degree; but its effect on the slope term is large.

This approximation is critical in the initial part of the residual-stress curve (small values of depth x) since the resultant stress is largely dependent on the slope. As the depth is increased, the resultant stress becomes more of a function of all three terms.

Conclusions

The experimental residual-stress distribution for the 316 stainless steel tends to justify the analysis made by Landau and Weiner. With a consideration of the lack of strain hardening assumed in the analysis and of the possible experimental error due to the scatter of data points, the agreement between analysis and experiment is good.

When a material undergoing no phase transformation is subjected to rapid cooling, the thermal conductivity of the material determines the magnitudes of the residual stresses to a large degree. The stress levels vary inversely with the thermal conductivity.

The experimental procedure utilized is valid where the change in strain with depth of cut remains positive. The procedure has the inherent limitation that some residual stress may be induced in the plate by the grinding process.

Acknowledgment

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APPENDIX

Rosenthal and Norton give as an equation relating residual stress, original stress and depth of cut,

$$\sigma_\alpha = \frac{(1-\alpha)}{2}\frac{dS_t}{d\alpha} - 2S_t + 3(1-\alpha)\int_{0.5}^{\alpha}\frac{S_t}{(1-u)^2}\,du \qquad \text{(A-1)}$$

which applies to one-half of the originally stressed plate. σ_α is the residual stress in the original stressed plate at depth α. S_t is the total stress removed from the upper surface of the plate by slicing the upper half of the original plate from the bottom. The value α is the depth of cut as measured from the bottom of the half-plate toward the upper surface.

Analytically, the differential of the stress S_t is shown to be:

$$dS_t = \frac{S_u'}{(1-u)}\,du + \frac{3S_u'}{(1-u)}\,du \qquad \text{(A-2)}$$

where S_u' is the stress removed by cutting an increment off the material at depth u. The first term in eq (A-2) represents the stress resulting from the change in axial loading of the plate; the second term presents the effects of a couple produced by the removal of an eccentric stress.

Since the stress indirectly measured by strain in this investigation is not the total change in stress from the plate in its original condition, but the stress change as measured from the remaining plate merely held flat, the first term is omitted from eq (A-2). Once the plate has been sliced, the axial loading is never replaced. Since dS_t is the sum of the terms from the effects of axial loading.

$$dS_t = dS_a + dS_m \qquad \text{(A-3)}$$

From eq (A-2)

$$dS_m = -3dS_a \qquad \text{(A-4)}$$

Therefore,

$$dS_t = -\frac{1}{3}\,dS_m + dS_m \qquad \text{(A-5)}$$

or

$$dS_t = \frac{2}{3}\,dS_m \qquad \text{(A-6)}$$

In eq (A-1), S_t is replaced by $^2/_3\,S_m$. Equation (A-1) becomes

$$\sigma_\alpha = \frac{(1-\alpha)}{3}\frac{dS_m}{d\alpha} - \frac{4}{3}S_m + (1-\alpha)\int_0^{\alpha}\frac{S_m}{(1-u)^2}\,du \qquad \text{(A-7)}$$

The third term integral limits are changed since an entire plate is considered.

For an infinite plate, components of stress parallel to the plane of the plate are equal. We have

$$\sigma_X = \sigma_Y \qquad \text{(A-8)}$$

It is assumed that no stresses exist in a direction normal to the plate. Thus,

$$E\epsilon_X = \sigma_X(1-\nu) \qquad \text{(A-9)}$$

E being the material modulus of elasticity and ν being Poisson's ratio.

Therefore,

$$\sigma_X = \frac{E\epsilon_X}{(1-\nu)} \qquad \text{(A-10)}$$

Equation (A-1) becomes, with a slight change in notation,

$$\sigma_\alpha = \left[\frac{E}{3}\frac{(1-\alpha)}{(1-\nu)}\frac{d\epsilon_b}{d\alpha} - \frac{4}{3}\frac{E\epsilon_b}{(1-\nu)} + \frac{2E(1-\alpha)}{(1-\nu)}\int_0^{d}\frac{\epsilon_b}{(1-u)}\,du\right] \qquad \text{(A-11)}$$

with ϵ_b being the change in strain as measured on the rear of the plate from the unrestrained to clamped position, or the strain induced in the rear of the plate by removal of material to depth α. The magnitude, slope and integral of the strain evident in eq (A-11) are obtained graphically from strain vs. depth-of-cut curves.