A Simple Procedure for Estimating High-temperature Low-cycle Fatigue

Author presents a simplified procedure for estimating the high-temperature reversed-strain-cycling fatigue characteristics of laboratory specimens

by S. S. Manson

Introduction

High-temperature metal fatigue is governed by many complex interactions. To take into account all the factors that enter into this process would be extremely difficult. Thus, as a first approach, it is well advised to seek simplifications that retain the basic elements of the problem while reducing the complexities as much as possible. In the following discussion, a highly simplified procedure for estimating the high-temperature reversed-strain-cycling fatigue characteristics of laboratory specimens will be presented. In addition, it will be shown that the fatigue lives of such specimens can be estimated with reasonable accuracy from a knowledge of the static tensile and creep-rupture properties at the temperature involved. It should be emphasized that the method is intended to give only first-approximation estimates as a guide to material selection. Final design of important equipment should be based on actual fatigue data generated under conditions which simulate as closely as possible those to be encountered in service.

Basis

To start, consider the simple approach that has been developed for estimating the low-cycle fatigue characteristics of materials subjected to reversed strain cycling at room temperature in order to see if it is possible to modify this approach to include the additional factors that have to be considered at high temperature. It has been found that the fully reversed-strain-cycling fatigue characteristics of materials at room temperature can be estimated by considering their tensile properties as measured in a monotonic tensile test—namely the ultimate tensile strength, the ductility and the elastic modulus. Room temperature is, in these cases, a temperature well below the creep range of the materials, or less than about half the absolute temperature at the melting point. Figure 1 illustrates the "method of universal slopes." The total-strain range is divided into its elastic and plastic components, and each component is plotted against cyclic life on log-log coordinates. Straight lines usually result for each of these components. The method of "universal slopes" prescribes that the slopes of these lines are assumed to be the same for all materials. The plastic components are taken to have an average slope value of $-0.6$, and the elastic components an average slope value of $-0.12$. Again, it should be emphasized that these values are

![Diagram](https://via.placeholder.com/150)

Fig. 1—Method of universal slopes for estimating axial fatigue life
Experimental Prediction by method of universal slopes

Cycles to failure, $N_t$

$0.00 \leq 0.4 \leq 0.8$ have been obtained for various materials for the plastic component, and values between $-0.08$ and $-0.16$ have been found for the elastic components. When average values of $-0.6$ and $-0.12$ are chosen for the plastic and elastic components, reasonable results are obtained.

Having decided on the slopes, it is then only necessary to determine the intercepts of the two straight lines. In general, it has been found that the property which most significantly governs the intercept of the plastic line is the ductility $D^*$. For the elastic line, the governing property for the intercept is $\sigma_u/E$, where $\sigma_u$ is the ultimate tensile strength and $E$ the elastic modulus. The total-strain range, given as the sum of the elastic and plastic components, thus becomes

$$\Delta \epsilon_t = \frac{3.5\sigma_u}{E} (N_p)^{-3.12} + D^* (N_p)^{-3.4}$$

(1)

as shown in Fig. 1.

The success of this approach applied to a number

of materials is indicated in Fig. 2 where the data for nine high-strength materials are presented together with the predictions based on the method of universal slopes. Correspondingly good agreement has been obtained for other materials.$^{1-3}$

The question arises, can one use the same procedure at high temperature? Can one use the tensile properties at high temperatures and predict in the same manner the high-temperature fatigue properties of the material? In general, it has been found that this approach almost always yields life predictions higher than those actually obtained by testing. The explanation for the unconservative results is, of course, very complex. To obtain a basis for an analytical approach, one might first consider the relation between high-temperature low-cycle fatigue and intercrystalline cracking.

High-temperature low-cycle failures are often attended by intercrystalline cracking as shown in Fig. 3. These are micrographs of the fatigue failures of two high-temperature alloys, L-605 and A-286. Intercrystalline cracking is evident in both materials when tested at $1200^\circ$ F. Numerous high-temperature tests on other materials have also been characterized by intercrystalline cracks. One
can, therefore, as a first approximation, examine this effect attending such intercrystalline cracks. From the standpoint of fatigue, such an intercrystalline crack may be considered as replacing the crack developed by reversed strain cycling during the first part of a fatigue test. Thus, the fatigue life for a given applied strain range at high temperature may be expected to be lower than that for the same strain range at a temperature below half the melting point, since at least some of the crack initiation (and, perhaps the crack propagation period as well) of the fatigue process is by-passed by the intercrystalline cracking. The amount of the reduction is probably a complex function of material and test conditions, but several simple approaches have been taken which yield ready answers, hopefully somewhere close to the correct ones.

In a first approach¹, the philosophy was to try to estimate how many cycles would be required to produce a crack in the ordinary fatigue process of smooth materials. Figure 4 shows the basis of this first approach. Experiments indicate that the number of cycles required to initiate a crack of a

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Fig. 3—Intergranular fatigue cracks at 1200°F

Fig. 4—Illustration of high-temperature crack-initiation hypothesis

\[
N_0 = N_f - 6.2N_f^{0.6}
\]

For \(N_f \leq 10^6\)

\[
N_0 = 0.85 \times 10^4
\]

\[
\Delta N = 0.15 \times 10^4
\]

Total life = \(1.0 \times 10^4\)

(a) Below creep range.

Assume \(N_s \ll N_f \sim 6.2N_f^{0.6}\)

At \(\Delta \varepsilon\) for which \(N_f \sim 10^4\)

Limiting case, \(N_0 = 0\)

\[
\Delta N = 0.15 \times 10^4
\]

Total life = \(0.15 \times 10^4\)

(b) In creep range.

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Fig. 5—Influence of frequency on elevated-temperature low-cycle fatigue behavior
few thousands of an inch in depth is about \( N_r = 0.85 \times 10^4 \) for a total life \( N_r = 10^4 \) cycles. In other words, about 85 percent of the life is used up in initiating the crack, and 15 percent of the life goes toward propagating the crack to failure. If, as shown in Fig. 4(b), it is assumed that intercrystalline cracking due to a "creep effect" occurs early in the life, and thereby by-passes the transcrystalline crack-initiation period, the total life is approximately equal to only the crack-propagation period. The life at high temperature is thus assumed to be reduced to only about 15 percent of the value it might have achieved had intercrystalline cracking not by-passed the crack-initiation period.

The above approach, together with others involving the concept of by-passing the crack initiation stage, are discussed more fully in Refs. 2 and 4. This concept has not been pursued but, instead, the approach designated as the "10% Rule," also described in these references has been adopted for further study. In this method, crack initiation and propagation are not regarded as functions of life; rather it is assumed that, on the average, only 10 percent of the life computed by the method of universal slopes will actually be realized at temperatures within the creep range.

Generally, it has been found that such a computation yields conservative results; that is, in most cases, it leads to a lower bound on life. However, there are cases in which even the "10% Rule" predicts lives that are higher than those actually achieved in test. This observation, together with the fact that the "10% Rule" approach inherently excludes the possibility of taking into account frequency effects, hold-time effects, mean load, etc., provide cause for one to seek further for simple methods of estimation that are not as limited.

The need for taking frequency effects into account is illustrated schematically in Fig. 5. The method of universal slopes, since it uses the values of ductility and tensile strength obtained in a conventional tensile test, is likely to yield a curve of strain range against cyclic life shown by the uppermost curve. This is so because, in the conventional tensile test, the strain rate is quite high. As the frequency, and consequently the strain rate, is decreased, both the ultimate tensile strength and ductility at high temperature are likely to be reduced. Thus, according to the method of universal slopes, the fatigue curves would in general be lowered by decreasing test frequency, as shown in Fig. 5. To take this into account, one could, of course, obtain tensile strength and ductility as a function of strain rate, and construct universal slope curves for the corresponding frequencies. This would involve a considerable amount of testing.

Another factor associated with frequency effect, but which in principle is equally related to hold-time and mean load, is shown in Fig. 6. Here the stress variation during a cycle is shown schematically for the case in which it is completely reversed; for other cases involving mean stress or hold-time, the stress variation may be more complex. At temperatures well below half the melting point of the material (and in the absence of chemical effects such as corrosion), the duration of the stress is not of great importance; the important factor is the magnitude of the stress. In the creep range, however, the duration of the stress becomes of paramount importance. If the frequency is high, many fatigue cycles can be traversed without exposing the material to creep effects due to the stress for any appreciable time. At very low frequencies, however, long exposure to stress can be accomplished in relatively few cycles. Since failure of the material results from the combined effect of both alternating strain and creep, it follows that the time of exposure to stress must enter into the analysis, as well as the number of cycles.

To account for the complex effects of varying stress, especially when compressive stresses are present, requires a more complete understanding of material behavior than is presently available. Thus, for a first approach, we omit the effect of the com-

\[ k = \text{EFFECTIVE FRACTION OF EACH CYCLE SPENT AT MAXIMUM TENSILE STRESS} \]

![Diagram](image-url)
pressive stress, and replace the entire stress pattern by one in which the stress is constant and equal to the maximum stress of the pattern it replaces. But the time for which it acts is chosen as only a fraction of the time required to complete the cycle. The replacement of one stress pattern by a simpler one is shown in Fig. 6. For the analysis which has been made thus far, the assumption that the constant stress acts for about 0.3 of the total cycle time has yielded reasonable results, but this assumption can readily be altered as more information becomes available. Once the stress and the time which it acts have been selected, the creep effect can be calculated.

The simplified analysis that has been adopted can be explained by the following. The "creep damage" effect is taken as the ratio of the time actually spent at stress to the time required to cause rupture at that stress value. Since the stress and temperature are presumably known, this rupture time can be obtained directly from the creep-rupture curve of the material. The "fatigue damage" effect is taken as the ratio of the number of cycles actually applied to the number that would be sustained in the absence of "creep effect" according to the method of universal slopes. Since the test frequency yields a definite relation between the time of the test and number of cycles sustained, a closed-form analytical expression for the number of cycles to failure can be obtained. This description is shown in the following derivation:

\[
\text{(Creep-rupture Damage) + (Fatigue Damage) = 1}
\]

\[
\left( t' \right) + \left( \frac{N_f'}{N_f} \right) = 1
\]

where:

\[
t' = \frac{k}{F} N_f'
\]

\[
t_f = A \left( \frac{\sigma_f}{1.75 \sigma_y} \right)^{1/m} = A (N_f)^{-0.12/m}
\]

hence:

\[
\left( \frac{kN_f'}{AF(N_f)^{-0.12/m}} \right) + \left( \frac{N_f'}{N_f} \right) = 1
\]

or:

\[
N_f' = \frac{N_f}{1 + k AF (N_f)^{m-0.12/m}}
\]

Here

\[
N_f' = \text{number of cycles to failure under combined fatigue and creep.}
\]

\[
N_f = \text{number of cycles to failure in fatigue, based on method of universal slopes, using ductility, ultimate tensile strength and elastic modulus from uniaxial tests at strain rates comparable to that experienced by the metal in the fatigue test. Where appropriate data are not available, use may be made of data from conventional tensile tests.}
\]

\[
k = \text{empirical constant, assumed to be 0.3, but adjustable as more information becomes available.}
\]

\[
m = \text{slope of straight-line creep-rupture curve on log-log coordinates, that is, representing curve by } \sigma_f = 1.75 \sigma_y \times A \text{ as shown in the insert in Fig. 7.}
\]

\[
A = \text{coefficient in creep-rupture relation.}
\]

\[
F = \text{frequency of cycling, cycles per unit time.}
\]

**Method**

In the foregoing section, two procedures for computing the lower bound on life have been outlined. They are:

1. By the "10% Rule." The universal slopes [eq. (1)] is first used to determine cyclic life, and this life is divided by a factor of 1.9.

2. By the combined creep and fatigue effect [eq. (2)].

The problem now is to determine which of the two equations to use, and how to interpret the results obtained from the computations. After considerable study of the data for many materials, the following conclusions can be drawn:

(a) Determine life by both methods (1) and (2) above, and use the lower of the two calculated values. Figure 7 is an auxiliary plot that minimizes the computations needed to determine which
value to use. It is merely necessary to determine the product, $A F$, and the slope, $m$, of the creep-rupture curve at the test temperature. If the point representing the coordinates lies above the curve, use $N_f$ according to eq (2). If it lies below the curve, use the "10% Rule.”

(b) The lower of the two values determined in (a) serves as a lower bound on estimated life.

(c) As an estimate of average or most probable life, use twice the lower bound determined in (b).

(d) As an estimate of the upper bound on life, use 10 times the lower bound determined in (b).

Thus, the method that has been adopted provides not only an estimate of the average life at a given set of test conditions, but also estimates of the upper and lower bounds within which the data are likely to fall. A study of how often these estimates have proven correct will be shown later. Also, although the method includes consideration of the "creep-fatigue" interaction, as shown by eq (2), it has been found that, in most cases encountered in the laboratory, the "10% Rule" is the applicable one, eliminating the need for direct consideration of creep. Some creep interaction is implicitly included when the universal slopes life value is divided by 10.

Application of the Method

The method has been applied to a large number of materials for which published fatigue data are available. The results are contained in two reports, and will be only briefly summarized here.

Figure 8 shows the results for the nickel-base alloy Nimonic 75 for temperatures from 1200 to 1800°F. The application of the criterion of Fig. 7, this case, indicated the "10% Rule" to provide the proper lower bound on life, and all the computations were made on this basis. Three curves are shown at each temperature. The lowest curve in each case provides the estimate of the lower bound, the heavier curve above it provides the estimate of the most probable value of life, and the upper-most curve the estimate of the upper bound on life. It is seen that, in all cases, the experimental data points obtained by Forrest and Armstrong fall between our estimated upper and lower bounds and, in most cases, the estimated average life lies quite close to the data points. Not all the data that have been analyzed agree as closely with the estimate as those shown in Fig. 8, but this figure is typical of the degree of agreement obtained in a large number of cases.

Figure 9 shows the results for several other nickel-base alloys wherein reasonably good results were again obtained. For the Astroloy and Udiment 700, the computations were again based on the "10% Rule," as indicated by the fact that all the curves shown are drawn as continuous lines. For the Inconel, however, the frequencies involved in the tests were quite low, and application of the criterion shown in Fig. 7 indicated the need for a creep rupture correction. The curves shown by the dotted lines were thus computed using eq (2) as the basis for the lower bound. It is seen that most of the data points fall close to the prediction for average life, but that, for two points, the data lie a little lower than the lower bound determined using the "creep effect" correction. The agreement is still quite acceptable, considering the nature of fatigue results, particularly at high temperatures.

While Figs. 8 and 9 show the results for only eight sets of data, numerous other sets of data have been analyzed, the details of which are described in Refs. 4 and 5. Shown below is a summary of these studies:

1. Number of different materials analyzed, 32.
2. Number of sets of data analyzed, 118 (979 data points).
3. Range of ductilities, 3 to 99 percent reduction of area.
4. Range of tensile strengths, 2 to 182 ksi.
Fig. 9—Comparison of computed and experimental fatigue behavior at high temperature, current nickel-base alloys

5. Range of temperatures and homologous temperatures, $T = 300$ to $2000^\circ$ F, $T_r T_m = 0.43$ to 0.88.
6. Range of frequencies, 0.0077 to 300 cpm.
7. Percent of data requiring use of $N_r$ instead of "10% Rule," 11 percent of sets, 7 percent of data points.
8. Percent of data points for which upper bound was adequate, 95 percent.
9. Percent of data points for which lower bound was adequate, 85 percent.
10. Percent of data points for which estimate of average life differed from experimental life by a factor less than each of several values, as shown in table below:

<table>
<thead>
<tr>
<th>Factor</th>
<th>Percent of Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>54</td>
</tr>
<tr>
<td>4</td>
<td>88</td>
</tr>
<tr>
<td>6</td>
<td>95</td>
</tr>
<tr>
<td>8</td>
<td>98</td>
</tr>
</tbody>
</table>

From the large number of materials and test conditions studied, and from the degree of agreement found between the test data and their estimated values, it can be seen that the proposed method can be a very useful tool in appraising the potential low-cycle fatigue performance of a material at high temperatures.

Concluding Remarks

Tempting as it is to conclude that the method gives results good enough for many engineering uses, it is perhaps of greater importance to emphasize some of the cautions involved in its use. It must first be emphasized that the data analyzed relate to constant-amplitude-strain cycling under constant temperature. While the method points to possible similar approaches for cases involving varying temperatures, varying strain amplitudes, complex stress histories, etc., it must be recognized that, in the present form, these deviations are not really included. It shows that the actual history of stress can be very important, emphasizing therefore the importance of hold-time, creep relaxation, and other factors contributing to the complexity of stress, but it approaches the question of stress determination only in the most elementary form. Certainly more detailed studies are needed. The method appears to be well able to place the high-temperature low-cycle fatigue behavior in the proper "ball park." However, it is not possible to predict in advance whether the estimated value will be higher or lower than the mean value. In some cases, although few in number, even the lower bound is not pessimistic enough. Thus, it would seem that while the method may be regarded as very good for screening materials, important material choices should be made on the basis of actual test from among the more promising materials. And, whenever possible, the complexities of stress and temperature history expected in service should be included in the test evaluation.

References