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Study of Residual Stresses in Linearly Varying Biaxial-stress Fields

Theory developed for the determination of relaxation strains around a hole is applied to the study of residual stresses at the toe of fillet welds

by H. V. Cordiano and V. L. Salerno

ABSTRACT—A theory has been developed for the calculation of relaxation strains effected by drilling a hole in a plate with a linearly varying stress field. With this theory, a technique was developed for the measurement of residual stress at the toe of tee-fillet welds. The above technique was employed for the measurement of residual stresses at the toe of tee-fillet welds in $1\frac{1}{2}$-in. HY-80 steel with the fillet in the as-welded, ground, shot peened, ground and shot peened, and mechanically peened condition. It was found that experimental data conform to the assumed theory, and that residual stresses in as-welded tee-fillet welds in both the transverse and longitudinal directions approach the yield strength of the steel. It was also found that residual stresses are reduced approximately 25 percent by grinding, 50 percent by shot peening and 50 percent by grinding and shot peening. Mechanical peening drastically affected residual stresses by converting high tension at the toe of the fillet weld to high compression of approximately the same magnitude.

List of Symbols

A = normal stress in x direction at origin, psi
B = normal stress in y direction at origin, psi
D = shear stress over entire plate, psi
E = modulus of elasticity
F = Airy stress function in Cartesian coordinates
Re = real part of complex function
a = radius of drilled hole
c = stress gradient applicable to σ_x, psi/in.
k = stress gradient applicable to σ_y, psi/in.
x, y = Cartesian coordinates
r, θ = polar coordinates
z = complex variable, $x + iy$
$\bar{z}$ = conjugate complex variable, $x - iy$
ϕ, X = complex stress functions
ν = Poisson's ratio
σ_x, σ_y = normal stresses in Cartesian coordinates, psi
τ_{xy} = shear stress in Cartesian coordinates, psi
σ_1, σ_2 = principal normal stresses, psi
σ_r, σ_θ = normal stresses in polar coordinates, psi
$(\epsilon_r)_R$ = relaxation strain in radial direction, in./in.

Introduction

Residual stresses developed during welding are considered to be an important factor, among others, contributing to the reduction in fatigue life. To alleviate this condition, a considerable amount of effort has been exerted to improve fabrication procedures by the introduction of grinding, shot peening, mechanical peening and combinations of these operations. A knowledge of the effects that these operations have on the magnitude and distribution of residual stresses would be of great value in the further development of improved fabrication techniques to increase fatigue life.

A realistic determination of residual stresses in structures, as affected by finishing operations, requires that the measurements be taken directly on the structure or on large structural elements simulating the part of the structure to be studied. The importance of large scale cannot be overemphasized here because residual stresses are directly affected by specimen size.

In addition to requiring a suitable specimen size, a valid determination of residual stresses requires that the techniques and procedures employed be appropriate for the conditions encountered. In recent years, a considerable amount of research has been directed toward the measurement of residual stresses in welded plates and structures by the hole-drilling method. This method has been exploited because of the availability of a variety of types and sizes of electric strain gages which provide for easy and accurate measurement of strain under various conditions. The hole-drilling method, sometimes referred to as the hole-relaxation method, entails the judicious placement of strain gages around a point at which residual stresses are to be determined, and measurement of the surface-relaxation strains which result from drilling a small hole at the point. From a knowledge of the relaxation strains, size of

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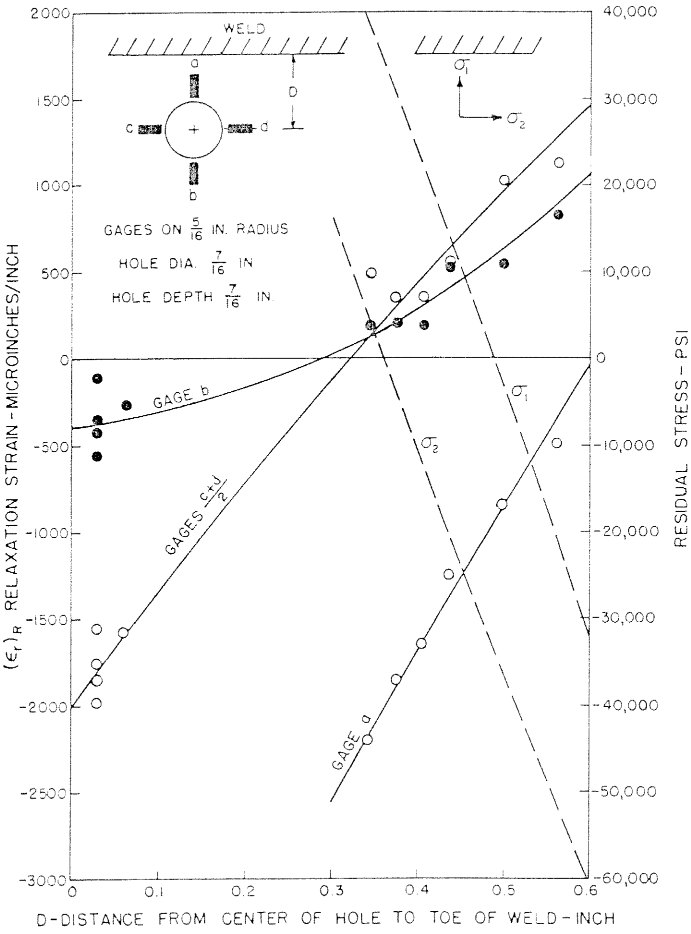

Fig. 1—Experimental data which formed the basis for assuming a linearly varying residual-stress field

hole and strain-gage layout, the magnitude of residual stresses at the point may be calculated. Initially, the method had been used on uniform biaxial-stress fields which did not vary with depth below the surface. Palermo[1] evaluated this hole-drilling technique on 27 plates 6-in. wide varying in thickness from $^1/_2$ to $1\,^1/_2$ in. and uniformly loaded through the thickness. He reported that residual stresses could be measured to an accuracy of approximately 5 percent if a $^7/_{16}$-in.-diam hole is drilled in the plate and radial relaxation strains are measured with 0.05-in. strain gages located on a $^5/_8$-in.-diam gage circle. Because the strain gages were small and as close to the edge of the hole as practical, full relaxation at the strain gages was obtained when the depth of hole reached one-half the hole diameter. Kelsey[2] used the hole-drilling method to determine residual stresses below the surface in a stress field that varied through the plate thickness. His approach is based on the premise that the incremental surface-relaxation strain for a corresponding incremental increase in hole depth is proportional to the magnitude of stress at that depth. His method is empirical and depends on experimental calibration.

Rendler and Vigness,[3] employing very small strain gages and small holes, developed the drilling technique to measure surface residual stresses in stress fields with large stress gradients. The method requires the determination of empirical calibration constants which could be used with different hole diameters if the laws of similitude are obeyed. The small volume affected by this method makes it particularly adaptable for measurement of localized residual stresses.

Preliminary experiments,[4] conducted at the U. S. Naval Applied Science Laboratory on the magnitude and distribution of residual stresses at the toe of tee-fillet welds in $1\,^1/_2$-in.-thick HY-80 steel, revealed that the stress gradient at the surface of the base plate normal and parallel to the weld was very large. This work is summarized in Fig. 1. The four-strain-gage array shown had been used to determine average residual stresses for the center of the hole. However, the distance D from the hole center to the toe of the weld was varied to provide the relaxation strains shown plotted. Data for gage a could not be obtained for D less than 0.34 in. because the edge of the hole approached the fillet weld and left no space for the strain gage. Data plotted for D equal to 0.03 in. were obtained with gage a omitted since the drilled hole was located so as to extend into the fillet. The large increase in relaxation strain, shown by gages a, c and d as the weld is approached, is noteworthy. These increases in ma nitude of the negative relaxation strain indicate increases in tensile residual stresses. Average residual stresses calculated from the plotted data are also shown in Fig. 1 as dashed curves. In making these calculations, a uniform biaxial-stress field was assumed.

Corresponding data from gages a and b were combined to give the average principal strain normal to the weld, and corresponding strains from gages c and d were combined to give the average principal strain parallel to the weld. It is seen that the curves representing the variation of average residual stresses with distance D are practically linear. These curves do not extend closer to the

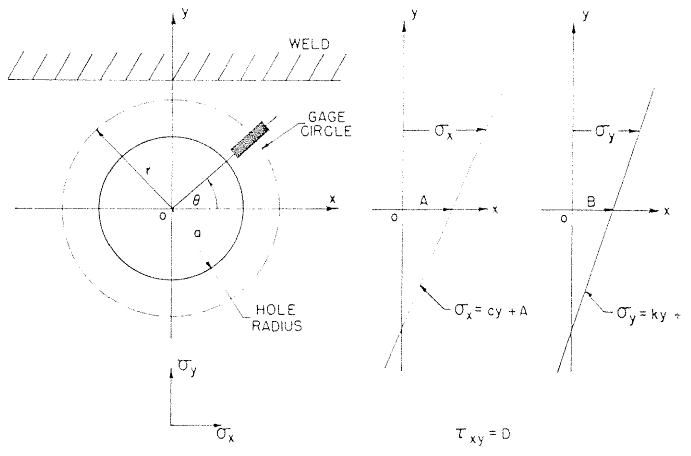

Fig. 2—Coordinate system and assumed stress distribution

weld than $D = 0.3$ in. since the plotted averages are stresses at the point where the center of the hole is located. For values of D greater than 0.5 in., the residual stresses are compressive. In this connection, measurements made in the base plate 6 in. away from the weld revealed surface residual stresses of $-44{,}000$ psi, indicating that the plate had initial surface compressive residual stresses of this magnitude prior to welding. Since the results showed a linear variation in both the transverse and longitudinal residual stresses, it was decided to develop an appropriate theory[4] for such a stress field and conduct related laboratory experiments.

Objective

The object of this paper is to:

(*a*) Present a brief summary of the theory developed for the determination of relaxation strains around a hole drilled in a linearly varying stress field.

(*b*) Present a technique developed for the application of this theory to the determination of residual stresses at the toe of tee-fillet welds.

(*c*) Present results obtained by use of this technique in the measurement of residual stresses at the tee-fillet welds in the as-welded, ground, shot peened, ground and shot peened and mechanically peened condition.

Theoretical Analysis

Referring to Fig. 2, it is assumed that residual stresses at the surface of a plate near a fillet weld are in a state of plane stress; that because of geometry the principal stresses σ_1 and σ_2 are in the x and y directions, parallel and normal to the weld, respectively; that these residual stresses are zero at some distance from the weld and rise linearly as they approach the weld; and that the stress field does not vary as the plate is traversed parallel to the weld. These residual stresses may be represented by

$$\begin{aligned} \sigma_x &= cy + A \\ \sigma_y &= ky + B \\ \tau_{xy} &= D \end{aligned} \tag{1}$$

where A, B, c, k and D are constants to be evaluated. It is noted that the stress distribution of eq (1), with a term $(-kx)$ added to the equation for shear stress, is derivable from the Airy stress function

$$F = \frac{Ay^2}{2} + \frac{Bx^2}{2} + \frac{Cy^3}{6} + \frac{kx^2y}{2} - D_{xy} \tag{2}$$

which satisfies the biharmonic equation

$$\nabla^4 F = 0 \tag{3}$$

However, retention of the term $-kx$ in the analysis and application of the results to the experimental data gave extremely high values for the stress gradients c and k and unrealistic values of the corresponding residual stresses. Because the actual conditions studied involved long tee-fillet welds, a shear stress which did not vary along the weld was considered more appropriate. Accordingly, the stress distribution of eq (1) which includes a constant shear stress D was selected for the analysis. Although this distribution does not satisfy stress equilibrium, it does provide a good fit to the experimental data.

When the unpenetrated infinite plate [containing the stress distribution corresponding to eq (1)] is penetrated by a circular hole, the stress and strain distributions around the hole become radically different from those in the unpenetrated plate. For convenience in calculating this new distribution, the stress distribution of eq (1) is considered in four parts, namely;

$$\sigma_x = cy \qquad \sigma_y = 0 \qquad \tau_{xy} = 0 \tag{4}$$

$$\sigma_x = 0 \qquad \sigma_y = ky \qquad \tau_{xy} = 0 \tag{5}$$

$$\sigma_x = A \qquad \sigma_y = B \qquad \tau_{xy} = 0 \tag{6}$$

$$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = D \tag{7}$$

Superposition of the effects of these four distributions will produce the distribution of eq (1).

In the solution of two-dimensional problems, the stresses may be derived from the Airy stress function [eq (2)] and satisfy the biharmonic eq (3). The stresses, derived from the Airy stress function, are

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} \qquad \sigma_y = \frac{\partial^2 F}{\partial x^2} \qquad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} \tag{8a}$$

It has also been shown that the Airy stress function $F(x,y)$ can be expressed as

$$F(x,y) = Re[\bar{z}\phi(z) + X(z)] \tag{8b}$$

where Re denotes the real part of the term in brackets and $\phi(z)$ and $X(z)$ are two independent, analytic functions of the complex variable $z = x + iy$.

If the functions $\phi(z)$ and $X(z)$ are known, the stress components σ_x, σ_y and τ_{xy} can be determined directly from these functions by

$$\sigma_x + \sigma_y = 4Re[\phi'(z)] = 2[\phi'(z) + \bar{\phi}'(\bar{z})] \tag{8c}$$

and

$$\sigma_y - \sigma_x + 2i\tau_{xy} = 2[\bar{z}\phi''(z) + X''(z)] \tag{8d}$$

where the prime and double prime denote the first and second derivatives, respectively, with respect to z. Details for the determination of the complex stress functions, and corresponding stress and strain distributions for the unpenetrated and penetrated plates, are given elsewhere.[4]

If the strain distributions for the unpenetrated and penetrated plate are expressed in polar coordinates, the difference in radial strain at each point is the relaxation strain. Thus, if a hole of given radius is drilled at some point 0, which is taken as the origin of coordinates, as shown in Fig. 2, and the change in radial strain near the edge of the hole is measured by means of a strain gage, the above

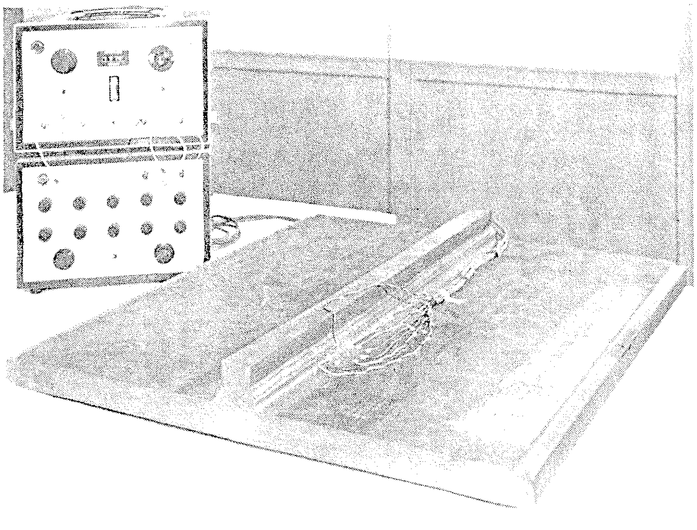

Fig. 3—Plate-type fatigue specimen of $1^1/_2$-in. HY-80 steel with ground tee-fillet weld and instrumentation for measuring relaxation strains by the hole-drilling method

theory shows that this change in strain, the relaxation strain, is given by the following equation:

$$
\begin{aligned}
E(\epsilon_r)_R = & -\frac{A+B}{2}(1+\nu)\frac{a^2}{r^2} - \\
& \frac{A-B}{2}\left[4\frac{a^2}{r^2} - 3(1+\nu)\frac{a^4}{r^4}\right]\cos 2\theta - \\
& \frac{cr}{4}(1+\nu)\frac{a^4}{r^4}\sin\theta + \frac{kr}{4}\left[(\nu-3)(\nu+1)\frac{a^2}{r^2}\right. \\
& \left. + \nu(1+\nu)\frac{a^4}{r^4}\right]\sin\theta - \\
& \frac{(c-k)r}{4}\left[(5+\nu)\frac{a^4}{r^4} - 4(1+\nu)\frac{a^6}{r^6}\right]\sin 3\theta - \\
& D\left[4\frac{a^2}{r^2} - 3(1+\nu)\frac{a^4}{r^4}\right]\sin 2\theta \qquad (9)
\end{aligned}
$$

where A, B, c, k and D are the constants of eq (1).

Since there are five constants, A, B, c, k and D, to be determined, at least five independent measurements of $(\epsilon_r)_R$ are to be made in five radial directions θ. It should be noted that the first two terms of eq (9) represent the known expression for the relaxation strain in a biaxial-stress field where A and B are the principal stresses. The next three terms of eq (9) represent the relaxation strain corresponding to the linear variation of the principal stresses with y where c and k are proportionality constants which give a measure of the strain gradient, and the last term represents the relaxation strain due to the shear stress D.

Specimens

Ten large-scale tee-fillet-welded, plate-type specimens fabricated from $1^1/_2$-in.-thick HY-80 steel were used in this work. Two specimens were in each of the as-welded, ground, shot-peened, ground and shot-peened, and mechanically peened condition. A typical specimen with ground fillet welds is shown in Fig. 3. The specimen is 32 by 29 in. and large enough to permit residual-stress measurements which are free from plate-edge effects. These specimens were identical to corresponding types used in a parallel laboratory large-scale fatigue program. All welding was conducted in accordance with NAVSHIPS 250-637-3. Mechanical peening was done with a pneumatic hammer and successive use of two blunted chisels. The point of the first chisel was a $^1/_4$-in.-radius hemisphere, while the point of the second was a $^1/_4$-in.-radius quarter cylinder.

Strain-gage Arrangement

The strain-gage arrangement employed in the measurement of radial relaxation strains by the hole-drilling technique is shown in Fig. 4. Eight strain gages spaced 45 deg apart are shown. Although five independent relaxation strains are required to determine the five constants of eqs (1), this eight-gage arrangement gives a uniformly weighted representation of the stress field around the hole. In addition, the resulting eight equations may be readily reduced to five through a combination which eliminates D and permits the use of the averages of symmetrically located gages e and f, c and d, and g and h. This gives a system of five equations and four unknowns, A, B, c and k which may be readily determined by the application of the method of least squares.

Method

The following techniques and procedures which are essentially in accordance with Palermo's[1] recommendations were followed in drilling the holes and measuring residual strains:

(*a*) The hole centers and the reference gage line for each of the fabrication conditions were located as shown in Fig. 4. For the as-welded and shot-peened fillets, the reference gage line was located $^1/_{32}$ in. away from the toe of the weld. For the ground, ground and shot-peened, and mechanically peened conditions, the reference gage line was located along a line representing the toe of the weld

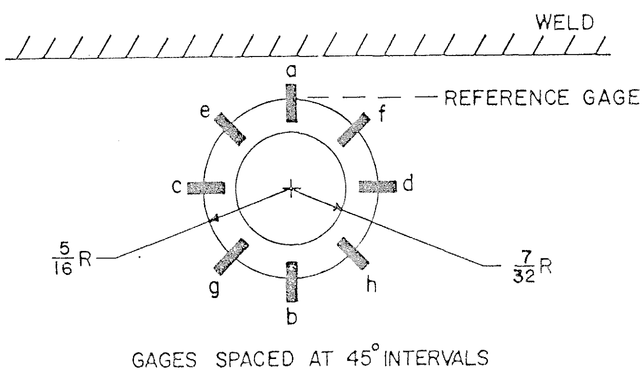

Fig. 4—Strain-gage arrangement at tee-fillet weld

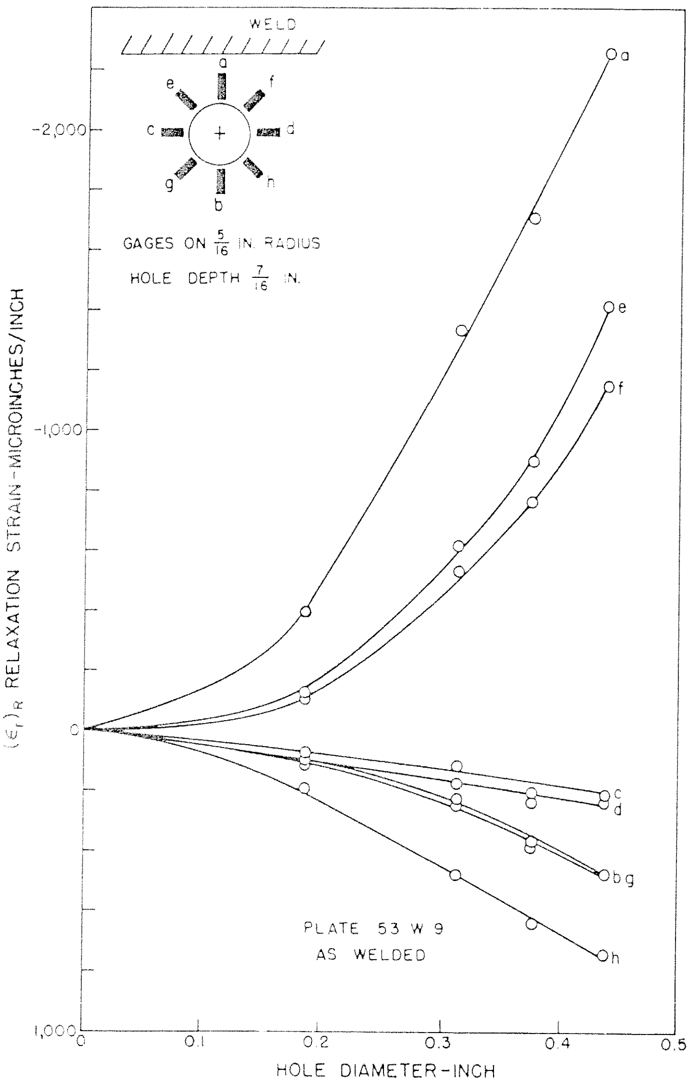

Fig. 5—Measured relaxation strain vs. hole diameter at various strain gages

prior to finishing by the respective fabrication procedures.

(*b*) Foil strain gages, type C6-1XIM50A, with a gage length of 0.050 in., carefully cemented at appropriate locations and protected with a coat of wax, were used for the measurement of relaxation strains.

(*c*) Strain gages were located on a $^5/_8$-in.-diam circle concentric with the drilled hole.

(*d*) Each set of gages was connected to a Baldwin switching and balancing unit and type N or Model 120 Baldwin strain indicator.

(*e*) A portable Bux magnetic-base drill, carefully aligned by means of centering pin in the chuck directly over the center of the hole, was used for drilling.

(*f*) Drilling was accomplished in increments using $^3/_{16}$, $^5/_{16}$, $^3/_8$ and $^7/_{16}$-in.-diam standard drills in succession while a stream of compressed air was directed towards the drill to blow the chips away, thereby avoiding damage to the gages. This procedure was followed in order to achieve the largest size hole without damage to the gages.

(*g*) Each hole was drilled to a depth of $^7/_{16}$ in.

(*h*) Measurements of relaxation strain were taken for all gage locations after each increment of drilling.

Relaxation strains were measured on each of the ten available plate-type specimens using the hole-drilling technique described above.

Results

Measured relaxation strains are shown plotted against hole diameter in Figs. 5, 6 and 7 for fillet welds in the as-welded, shot peened after grinding, and mechanically peened condition. Results for ground fillet welds and shot-peened fillet welds were similar to those shown in Figs. 5 and 6, respectively, but are not shown. It should be noted that for the as-welded, ground and mechanically peened specimens, all gages showed an increase in the magnitude of the relaxation strain as the hole diameter was increased. Both positive and negative relaxation strains were obtained. In general, the positive values indicate release of compressive

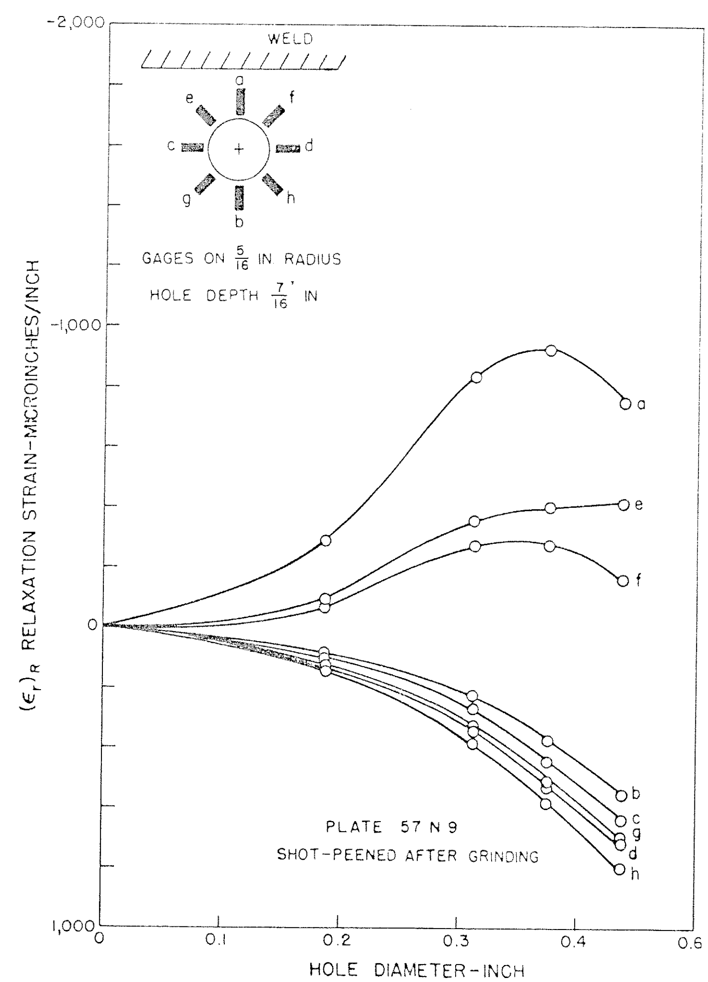

Fig. 6—Measured relaxation strain vs. hole diameter at various strain gages

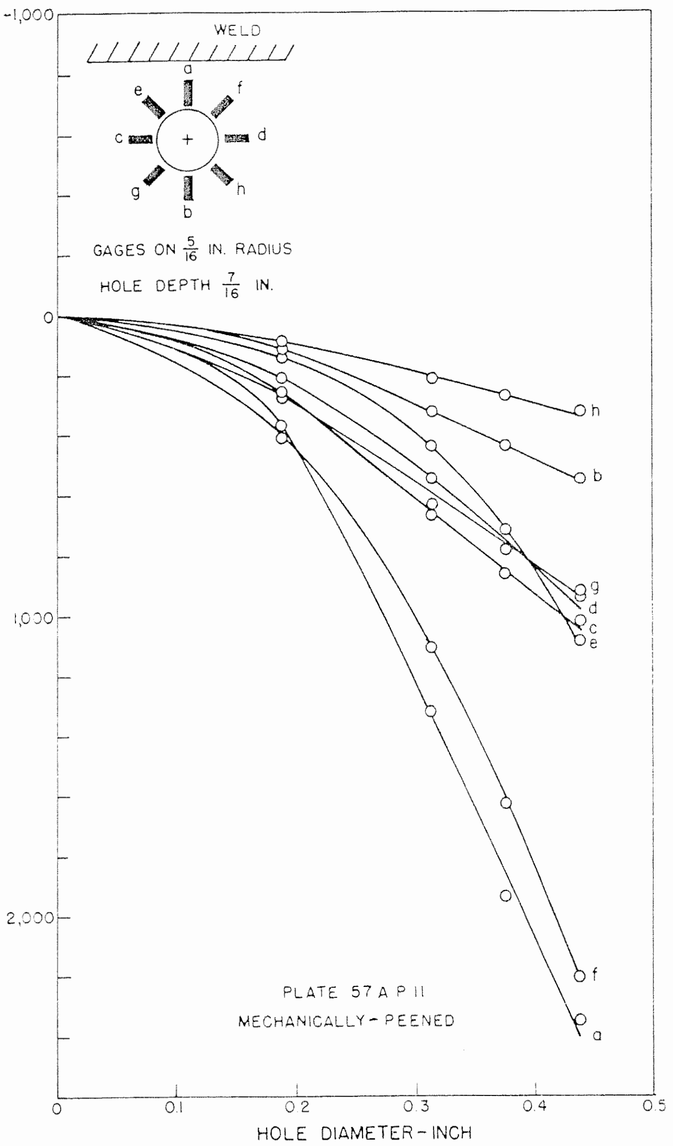

Fig. 7—Measured relaxation strain vs. hole diameter at various strain gages

residual stresses and the negative values indicate release of tensile residual stresses. In the case of the shot-peened and shot-peened after grinding specimens, the measured tensile residual strains rose to a maximum and then dropped off as the hole diameter was increased. The wide difference in the nature of relaxation strains measured on as welded and mechanically peened specimens is noteworthy.

Equation parameters A, B, c, k and D calculated from measured relaxation strains at a hole diameter of 0.438 in. by means of eq (9) are given in Table 1 for the ten plates studied. Also tabulated are the residual stresses at the toe of the welds as determined from eqs (1).

Discussion

A discussion of the results given in Table 1 is made with full cognizance of the following limitations:

(a) The theory is based on a plane-stress field which is uniform through the thickness of the plate, whereas it is expected that the actual residual-stress field varies through the thickness.

(b) The theory is developed for stresses within the elastic range, whereas the magnitude of radial stresses measured indicates that the tangential stresses at the very edge of the hole are in the plastic range.

(c) The results have not been corrected to offset these limitations.

The assumption of a linearly varying residual-stress field near a fillet weld appears to be supported by a comparison of measured and calculated radial relaxation strains. This comparison is given in Fig. 8 for as-welded and mechanically peened specimens. The curves marked theoretical are a plot of eq (9) which was derived from the stress field of eqs (1). The values of A, B, c, k and D used in eq (9) are given in Table 1. It is noted that the experimental data, shown as open circles, fall very close to the corresponding theoretical curve for the as-welded specimen and fairly close for the

TABLE 1—RESULTS DETERMINED FROM MEASURED RELAXATION STRAINS

		Equation parameters					Residual stress at toe of weld*	
Condition of tee-fillet weld	Plate no.	A, ksi	B, ksi	D, ksi	c, ksi/in	k, ksi/in	σ_x, ksi	σ_y, ksi
As-welded	53W9	−1	32	0	260	268	88	124
	54W10	3	34	−1	278	240	99	117
Ground	57G9	−1	27	−3	279	200	86	90
	58G10	−15	21	1	169	222	38	90
Shot peened	53P9	−23	2	−1	127	134	21	48
	54P10	−28	0	−1	170	136	30	47
Shot peened after grinding	57N9	−23	−1	−2	157	135	25	41
	58N10	−11	2	−3	222	120	58	40
Mechanically peened	57AP10	−48	−54	2	−27	−77	−56	−78
	57AP11	−49	−63	−3	−110	−173	−83	−117

* All values above 80 ksi are in the plastic range and therefore represent "equivalent elastic" stresses.

mechanically peened specimen. In the latter case, the residual stresses were transformed from high tension to high compression, by the peening operation. This large change in residual stress by a manual operation would tend to introduce some nonuniformity in the resulting residual stresses. The approximate linearly varying compressive residual-stress field arises from a combination of high compressive residual stresses in the peened area and the lower compressive residual stresses already existing in the adjacent base-plate surface prior to welding. Experimental data for the ground, shot-peened and shot-peened after grinding specimens also fell very close to the theoretical curves.

From Table 1 it is observed that residual stresses after shot peening are indicated as tensile. Since shot peening normally puts the surface under compression, a question arises as to significance of the experimental result. As was noted in Fig. 6, the negative relaxation strain increased in magnitude to a maximum with increasing hole size and then fell off. It appears that, for small hole diameters, the subsurface tensile residual stresses have a significant effect on the measured relaxation strain and that, for the larger diameters, the surface residual stresses predominate. It is apparent in this case that the residual stresses were not uniform through the thickness that was drilled.

The parameters A and B of Table 1 represent the average biaxial residual stresses at the point represented by the hole center. These values are identical to those that would have been calculated from the experimental data if a uniform biaxial-stress field were assumed. The residual stresses at the toe of the weld determined on the basis of a linearly varying stress field and listed in Table 1 are significantly higher than the stresses A and B determined on the basis of a uniform stress field. These wide differences justify the application of the proposed theory and techniques to residual-stress fields with sharp gradients. The shear stress D in Table 1 is quite low and does not significantly affect the symmetry of the plotted relaxation strain curves of Fig. 8. In addition, the close fit of the experimental data to these curves supports the assumption that principal stresses are parallel and normal to the tee welds.

The results of Table 1 indicate that surface residual stresses at the toe of as-welded tee-fillet welds in $1\frac{1}{2}$-in.-thick HY-80 steel plate are at least equal to the yield strength of the steel. The lowest residual stress shown is 88,000 psi parallel to the weld and the highest shown is 124,000 psi normal to the weld. Obviously, since the ultimate strength of the HY-80 steel is 105,000 psi, these high values of residual stress indicate that large strains associated with plastic behavior were developed during the drilling process.

The results of Table 1 also show that:

(*a*) Grinding the fillet weld to contour reduced residual stresses moderately. The values of 86,000 and 38,000 psi for σ_x appear far apart. This difference may be due to difficulty in locating the critical section after grinding for accurate placement of strain gages.

(*b*) Shot peening reduced residual stresses significantly. Although this process is intended to put surface metal in compression, the results indicate tensile residual stresses. It appears that high-tensile residual stresses immediately below the surface were effective in controlling the final state of stress at the surface.

(*c*) Shot peening after grinding appears to have had about the same net effect on the magnitude of residual stresses as shot peening alone. It had been expected that the combination would result in a greater reduction. However, various factors, such as actual initial residual stresses, location of ground contour, selection of critical section for strain gages, severity of shot peening, etc., affect the final result.

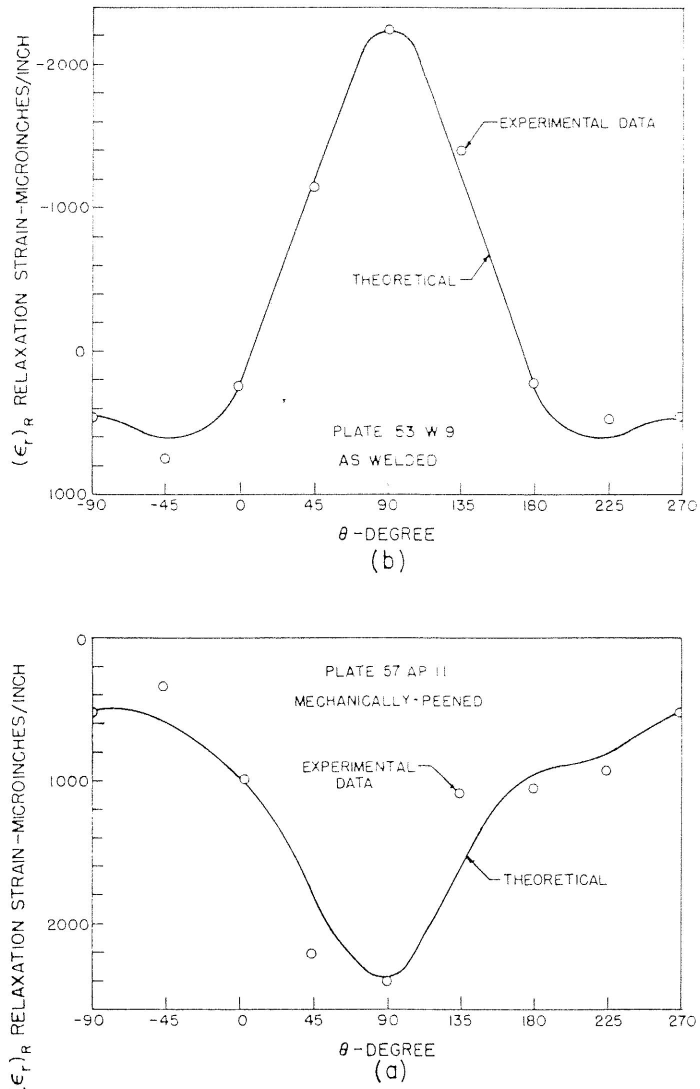

Fig. 8—Variation of radial relaxation strain with angle around drilled hole

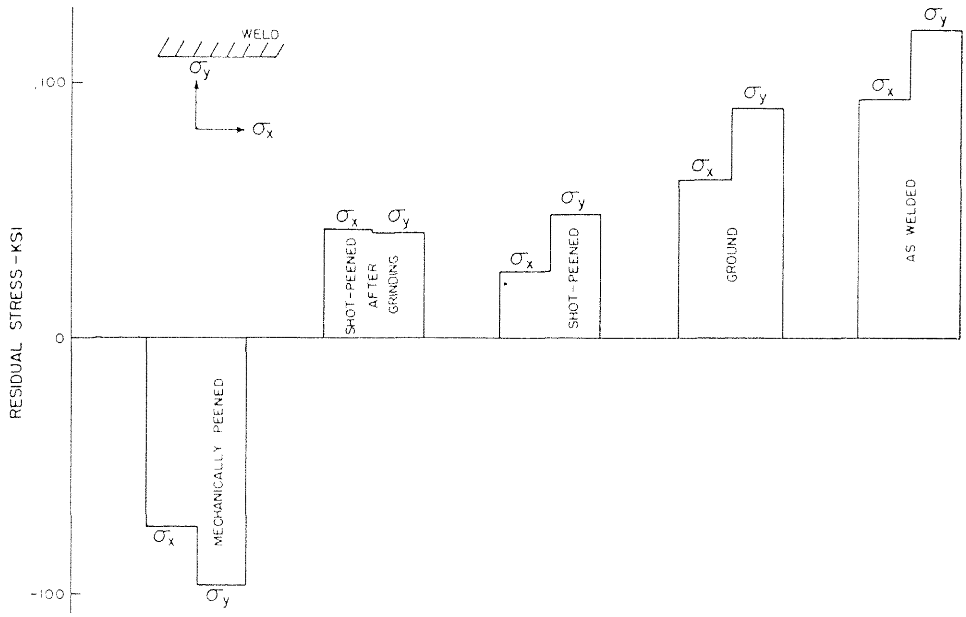

Fig. 9—Average residual stresses at toe of tee-fillet welds as affected by various fabrication processes

(*d*) Mechanical peening drastically affected residual stresses by converting high tension at the toe of the fillet weld to high compression of approximately the same magnitude. The extensive plastic deformation associated with mechanical peening is the sole cause for the large reversal in residual stresses.

Conclusions

An appropriate theory has been developed for the determination of surface residual stresses over a limited area in linearly varying biaxial stress fields. This theory is also applicable to uniform stress fields and therefore may be generally applied.

Techniques for the application of the theory to the measurement of residual stresses at the toe of tee-fillet welds have been developed. Hole centers for drilling may be conveniently located and residual stresses over limited areas may be readily determined. Precautions must be taken in interpreting results if residual stresses vary through the thickness.

Experimental results tend to confirm the approximate theory and the assumption of a linearly varying stress field at tee-fillet welds. The above conclusions are based on determinations made with $^7/_{16}$-in.-diam drilled holes with strain gages placed on a $^5/_8$-in.-diam gage circle. Smaller holes may require calibration.

Residual stresses at the toe of as-welded tee-fillet welds in $1^1/_2$ in.-thick HY-80 plate are tensile in both the transverse and longitudinal directions and approach the yield strength of the steel. Results reported in Table 1 are higher than the yield strength because of the large strains associated with stress concentration in the tangential direction at the edge of the hole and the resulting plastic deformation.

The gross effects of fabrication processes on residual stresses at tee-fillet welds are depicted by the bar graph, Fig. 9. The graph shows that:

(*a*) Grinding reduces the surface tensile residual stresses in tee-fillet welds by roughly 25 percent.

(*b*) Shot peening reduces the surface tensile residual stresses in tee-fillet welds by roughly 50 percent.

(*c*) Shot peening after grinding reduces surface tensile residual stresses by roughly 50 percent.

(*d*) Mechanical peening reverses the surface residual stresses in tee-fillet welds from high tensile to high compressive residual stresses of approximately the same magnitude.

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