Predicting Failures with Conducting-polymer Fatigue-damage Indicators

Paper deals with indirect approach for developing the relation between the output from the fatigue-damage indicator and the fatigue performance of the component

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ABSTRACT—A comparison of the response of a conducting-polymer fatigue-indicating device with the fatigue behavior of a number of engineering materials shows an excellent match for Al 6061-T6 and a fair match for 1018 and 4130 steels. When the indicator is adequately matched to a material, the onset of fatigue failure can be predicted by a critical resistance change in the indicator. For more-complex fatigue exposures involving multi-level loads, the indicator appears to predict failure with a critical value of \( \Delta R/R \) which is a constant for a specified material.

Introduction
The prediction of fatigue damage in a machine component or a structure requires a detailed knowledge of the load-time history, properties of the material and an adequate cumulative-damage relationship. This prediction is extremely difficult as a large number of parameters affect fatigue life and the exact load-time history is rarely ever completely known.

An alternate approach was advanced by Harting in which a resistive transducer was attached to the machine component and its electrical output is related to fatigue life. The transducer adapted by Harting was a modified foil-type strain gage in which the constantan foil was annealed. More recently Dally and Panizza have introduced a different resistive transducer to be used in predicting fatigue life. This transducer is fabricated from a conducting polymer consisting of large graphite particles in a highly plasticized epoxy matrix.

In both instances, the authors refer to the resistive transducers as fatigue-damage indicators which implies that a relation exists between the damage induced in the material of the component and the output of the gage.

There are two approaches which can be employed to develop the relation between the output from the fatigue-damage indicator \( \Delta R/R_i \) and the fatigue performance of the component. In the direct approach, the transducer is properly located on the component and \( \Delta R/R_i \) is established as a function of time while the component is operating under actual conditions until fatigue failure occurs. This direct calibration procedure gives the critical value of \( \Delta R/R_i \) and a close approximation of relation between \( \Delta R/R_i \) and the remaining fraction of the life of the component.

When direct calibration is not possible, it becomes necessary to utilize an indirect approach relating the output from the fatigue-damage indicator to the cycle-strain history and the fatigue characteristics of the materials employed in the component. This paper treats this indirect approach by first considering the results of Coffin and Manson to analytically describe the fatigue response of the materials. Next, another set of analytical expressions are derived to give the resistive response of a fatigue-damage indicator fabricated from a conducting polymer as composition D. Several materials are considered and it is shown that the indicator can be matched to the material so that the critical value \( \Delta R/R_i \) is essentially a constant over a significant range of fatigue lives when the fatigue loading is of constant amplitude. Finally, variable-amplitude fatigue exposure is treated and using Miner's damage theory it is again shown that the critical output \( \Delta R/R_i \) can be considered essentially constant.

Fatigue Properties of Materials
The fatigue behavior of metallic materials can be divided into two regions, namely the low-cycle and the high-cycle regions. In the low-cycle region (with \( N \approx 10^3 \)), the material exhibits a nonlinear stress-strain behavior and the strain range is the parameter which is related to fatigue life. In the high-cycle region (with \( N \approx 10^6 \)), the strains usually remain below the yield point and are essentially proportional to stresses.

The fatigue life of many metallic materials in the low-cycle region can be approximated by a relation due to Coffin which is written as:

\[ N^{0.2} \Delta e = C \]  

(1)
where

\[ N_f = \text{number of cycles to failure} \]
\[ \Delta \sigma_p = \text{the plastic-strain range} \]
\[ k = \text{a power exponent equal to 0.5} \]

and

\[ C = - (1/2) \ln (1 - RA) \]  

where \( RA \) is the reduction in area for a given material.

The range of application of eq (1) can be extended by considering a modification (see Ref. 4) which introduces the total strain range as:

\[ \Delta \varepsilon_t = \Delta \varepsilon_p + \Delta \varepsilon_e = \frac{C}{N_f^{0.5}} + S_o \]

where

\[ S_o = \text{the endurance limit} \]
\[ E = \text{the modulus of elasticity} \]

and the strain ranges \( \Delta \varepsilon_t, \Delta \varepsilon_p \) and \( \Delta \varepsilon_e \) are defined in the cyclic stress-strain diagram shown in Fig. 1(a).

The total stress range based on an elastic behavior can be written as:

\[ 2S = E \Delta \varepsilon_t \]  

and combining the results of eq (4) with eq (3) gives

\[ S = \frac{EC}{2N_f^{0.5}} + S_o \]  

(5)

Although the stress \( S \) is fictitious, as indicated in Fig. 1, Coffin and Tavernelli\(^4\) have demonstrated that the prediction of S-N curves by eq (5) closely fit experimentally determined data, provided \( N_f \leq 10^6 \) cycles.

Manson\(^2\) also developed an expression to predict fatigue life. This expression shown below:

\[ \Delta \varepsilon_t = 3.5 \sigma_u N_f^{-0.12}/E + D^{0.6}N_f^{-0.6} \]  

(6)

where

\[ \sigma_u = \text{the ultimate tensile strength} \]
\[ D = \text{the ductility ln} \{1/(1 - RA)\} \]

utilizes results which can be obtained by a simple tension test.

The results of eq (6) correspond to a fatigue condition in which the stress or strain was completely reversed so that the mean strain was zero. There are two different cases involving a mean strain depending upon whether the fatigue loading produces a plastic or elastic response. The first case is illustrated in Fig. 2(a) where a specimen is cycled between the cyclic stress-strain range of application of eq (1) can be extended.

In the second case, the specimen is subjected to a cyclic elastic strain with a mean stress is zero. In this case, the mean stress is zero. When the altering strain is elastic, a mean stress can be sustained and the modified Goodman law can be applied to determine the \( \Delta \varepsilon_t - N_f \) relationship as illustrated in Fig. 3. Consider a loading condition with an alternating stress \( \sigma_a \) and a mean stress \( \sigma_m \). Then, from the modified Goodman line\(^6\) and proportional triangles, it is evident that expressions can be derived for the allowable stress or strain range. For example, when the conditions corresponding to a stress state \( A \) are fluctuating tension such as \( \sigma_m = \sigma_a \), then it can be shown that

\[ \Delta \varepsilon_t = \frac{2(\sigma_a)A}{E} \left[ \frac{\sigma_u}{\sigma_u + (\sigma_a)B} \right] \]  

(7)

where \( (\sigma_a)B \) is defined in Fig. 3 and, by using eq (6), it is clear that

\[ (\sigma_a)B = E \Delta \varepsilon_t/2 = 1/2(3.5 \sigma_u N_f^{-0.12} + E^{0.6} N_f^{-0.6}) \]

Substituting this relation into eq (7) gives:

\[ \Delta \varepsilon_t = \frac{(3.5 \sigma_u N_f^{-0.12}/E) + D^{0.6}N_f^{-0.6}}{1 + (1/2 \sigma_u)} \left(3.5 \sigma_u N_f^{-0.12} + E^{0.6} N_f^{-0.6}\right) \]  

(8)

for the strain range for the special case when \( \sigma_m = \sigma_a \). According to Langer,\(^5\) eq (8) will be valid providing

\[ \sigma_a + \sigma_m = 2\sigma_a < \sigma_y \]

where \( \sigma_y \) is the yield stress.

For nonmetallic materials such as glass-reinforced
plastics, it is also possible to relate fatigue life to strain range. Dally and Agarwal showed that the fatigue life, \( N_f \), of a glass-reinforced plastic known commercially as Scotchply 1000 could be related to the strain range by

\[ N_f^{0.017} \Delta e = 0.0203 \]  

(9)

where the strain was cycled from zero to a maximum tension.

These analytical expressions described the fatigue behavior of many engineering materials under different types of cyclic loads. By combining these relations with the characteristic output of a fatigue-damage indicator, it is possible to essentially calibrate the gage by establishing the critical value of \( \Delta R/R_1 \) associated with fatigue failure.

**Determination of the Critical Resistance of the Fatigue-damage Indicator**

A conducting-polymer fatigue-damage sensor undergoes a monotonic change of resistance when bonded to a component fabricated from a specified material and subjected to a prescribed cyclic tensile strain. For instance, a typical sensor made from graphite flakes and a flexibilized epoxy matrix (see Ref. 2) exhibits the \( \Delta R/R_1 \) vs. \( N \) response curves presented in Fig. 4. The sensor, a single conductor about \( \frac{1}{2} \)-in. long, is deployed between two copper tabs on a strain-gage terminal strip. The cross section of the conductor has the shape of a segment of a circle with a width of about 0.1 in. and a thickness of about 0.025 in.

The response of the sensor can also be described analytically as:

\[
\log (\Delta R/R_1) = A(\Delta e) + B(\Delta e) \log N \tag{10}
\]

for \( N \geq 100 \)

The coefficients \( A \) and \( B \) are functions of the strain range which can be expressed in a relatively simple form:

For 
\( \Delta e \leq 0.4 \) percent

\[ A(\Delta e) = -0.726 \Delta e^{-0.282} \]

\[ B(\Delta e) = 0.107 \Delta e^{-0.128} \tag{11} \]

For 
\( \Delta e > 0.4 \) percent

\[ A(\Delta e) = -0.852 \Delta e^{-0.254} \]

\[ B(\Delta e) = 0.124 \Delta e^{-0.272} \tag{12} \]

Results obtained from eqs (10), (11) and (12) are presented in Fig. 5 in the form of the straight lines on the log-log graph. The experimental data points for \( \Delta R/R_1 \) obtained at different strain levels are also shown to indicate the fit of the analytical representation given by eq (10).

The critical resistance of a fatigue-damage indicator can be established by using eq (10) together with the appropriate relation given in the section on "Fatigue Properties of Materials" to describe the
fatigue behavior of a particular material. In general, the critical resistance will depend upon the material, the loading conditions and the fatigue life. For example, consider a mild steel (1018 annealed) which typically exhibits $S_f = 20,000$ psi and $RA = 0.65$; then, from eq (2), $C = 0.525$ and from eq (3):

$$\delta_t = 0.525/N^{0.5} + (4/3) \times 10^{-3}$$

Consider an intermediate life with $N_f = 2 \times 10^5$, then $\delta_t = 0.011$. The critical resistance of fatigue-damage indicator is then established from eqs (10) and (12) as $(\Delta R/R_i)_CR = 0.391$. Other values of $(\Delta R/R_i)_CR$ were established for the fatigue-damage indicator with $\delta_t$ varying between 1.8 and 0.5 percent ($10^3 < N_f < 2 \times 10^6$). It was found that $(\Delta R/R_i)_CR$ ranged from 0.41 to 0.38 over the life interval from $10^2$ to $2 \times 10^5$. As this variation is small, the critical resistance for 1018 C steel can be treated essentially as a constant with $(\Delta R/R_i)_CR = 0.4$ over the specified life interval. The adequacy of the fit of the constant gage response is shown in Fig. 6 where $\delta_t$ is shown as a function of $N$. Here the curve of the $(\Delta R/R_i)_CR = 0.4$ nearly coincides with the $\delta_t = N_f$ curve for 1018 C steel. For strain ranges smaller than 0.5 percent, with $N_f > 2 \times 10^4$, the critical resistance decreases as the life $N_f$ increases to infinity. In this life interval, it is necessary to establish the cyclic strain on the component to determine the critical resistance of the fatigue-damage indicator.

As a second example, consider a component fabricated from annealed 4130 steel with typical properties, $RA = 0.6$, $\sigma_i = 95,000$ psi and $D = 0.91$. Substitution of these values into eq (6) gives the strain range-life relation:

$$\delta_t = 11.08 \times 10^{-4} N_f^{-0.12} + 0.945 N_f^{-0.46}$$

The values of $\delta_t$ obtained from this equation change from 2.0 to 0.37 percent as $N_f$ varies from $10^3$ to $10^5$ cycles. Substitution of these results into eqs (10), (11) and (13) give values of $(\Delta R/R_i)_CR$ which vary from 0.42 to 0.37. Again, it is apparent that $(\Delta R/R_i)_CR$ can be treated as a constant equal to 0.40 over the life interval $10^3 < N_f < 10^5$ where the critical response of the damage indicator is essentially independent of the strain range. For a fatigue life $N_f > 10^5$, the value of $(\Delta R/R_i)_CR$ decreases to about 0.30 as $N_f \rightarrow \infty$, and is dependent upon the magnitude of strain range $\delta_t$. The match between the gage response and the fatigue response of 4130 is illustrated graphically in Fig. 7.
Improving the Match Between the Indicator and the Material

For certain materials, the match between the response of the indicator and response of the material to a fatigue exposure does not lead to a constant value of $(\Delta R/R)_{CR}$. In other instances, $(\Delta R/R)_{CR}$ is beyond the range of applicability of the indicator made from composition D which is limited to $(\Delta R/R)_{MAX} = 0.40$. The aluminum alloy 2024-T3 and the long-fiber-fortified nylon are materials which do not match well with the gage. This fact is illustrated in Fig. 8, where a comparison of the response curves is made.

The match between the output of the indicator and the fatigue-failure characteristics of the material may be improved by reducing the strain to which the indicator is exposed. The strain reduction may be accomplished in several ways. The simplest reduction is by rotating the gage (providing it is long and slender) through an angle $\theta$ in the strain field. Then, from the equations of strain transformation, the strain range is given by:

$$\Delta e_{II} = \Delta e_{L} \cos^2 \theta + \Delta e_{T} \sin^2 \theta$$

(13)

where $\Delta e_{II}$ = total strain range along the longitudinal axis of the gage

$\Delta e_{L}$ and $\Delta e_{T}$ = total range in the principal strains

It is also possible to reduce the strain on the indicator by mounting it at a location on the specimen which is noncritical. This location will undergo the same strain history as the location of the maximum strain, except that the magnitude of the strain will be reduced (providing yielding does not occur at the location of the maximum strain).

When the strain on the indicator is reduced, the value of $(\Delta R/R)_{CR}$ is also reduced and the match between the material response and the indicator output is markedly improved. Referring to Fig. 8, it shows that a reduction in strain with aluminum alloy 2024-T3 by a factor of two gives an excellent match with $(\Delta R/R)_{CR} = 0.32$ for $10^3 < N_f < 10^5$. Similarly, a reduction in strain on the indicator by a factor of three, for the long-fiber-fortified nylon, gives a close match over the interval $10^3 < N_f < 10^5$ with $(\Delta R/R)_{CR} = 0.32$.

It is clear in both of these cases that $(\Delta R/R)_{CR}$ could be treated as a constant for the specified material, regardless of the strain range at which the indicator is cycled.

Indication of Cumulative Damage

The applicability of the fatigue-damage indicator to predict fatigue failures of different materials subjected to fatigue cycling at constant amplitude has been demonstrated. However, the important application of the device on components subjected to more complex load-time histories must be demonstrated. To treat this very complex application, assume that the output of the indicator and the fatigue response of the material are closely matched so that $(\Delta R/R)_{CR}$ is a constant for the material. Next, assume that the material damaged follows Miner's law:

$$d = \frac{n_1}{N_{1f}} + \frac{n_2}{N_{2f}} + \ldots = 1$$

(14)

where $n_1$ and $n_2$ = number of cycles of exposure at strain ranges $\Delta e_{1}$ and $\Delta e_{2}$, respectively

$N_{1f}$ and $N_{2f}$ = number of cycles required for failure at $\Delta e_{1}$ and $\Delta e_{2}$, respectively

With this approach, a complex cyclic loading can be approximated by successive blocks of constant-amplitude loading. The damage in the specimen is related to the cycle ratio $n_1/N_{1f}$ and, when the sum of these cycle ratios amounts to one, failure occurs.

In treating the response of the fatigue-damage indicator to a loading with variable amplitude, a two-level lock loading was considered. The resistance change $\Delta R/R_1$ associated with the first block of $n_1$ cycles at a strain $\Delta e_1$ obtained from eq (10) as

$$\Delta R/R_1 = \log^{-1} \left[ A(\Delta e_1) + B(\Delta e_1) \log (n_1) \right]$$

(15)

This resistance change could also be produced by a different number of cycles at the strain $\Delta e_2$. This equivalent number of cycles $n_{2e}$ is obtained from eq (10)

$$n_{2e} = \log^{-1} \left[ \frac{\log(\Delta R/R_1) - A(\Delta e_2)}{B(\Delta e_2)} \right]$$

(16)

The second block of $n_2$ cycles at the strain $\Delta e_2$ further increases the resistance change to

$$\Delta R/R_{1,2} = \log^{-1} \left[ A(\Delta e_2) + B(\Delta e_2) \log (n_2 + n_{2e}) \right]$$

(17)
TABLE 1—CRITICAL RESPONSE OF THE DAMAGE INDICATOR FOR FIVE DIFFERENT FATIGUE LOADINGS

<table>
<thead>
<tr>
<th>Δε (%)</th>
<th>Nf</th>
<th>Nfr</th>
<th>n1</th>
<th>n2</th>
<th>(ΔR/R1)</th>
<th>m2</th>
<th>(ΔR/R1)m2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.45</td>
<td>0.77</td>
<td>550</td>
<td>4000</td>
<td>275</td>
<td>2000</td>
<td>33</td>
<td>2260</td>
</tr>
<tr>
<td>1.45</td>
<td>0.77</td>
<td>550</td>
<td>4000</td>
<td>550</td>
<td>4000</td>
<td>550</td>
<td>3780</td>
</tr>
<tr>
<td>0.77</td>
<td>0.77</td>
<td>550</td>
<td>4000</td>
<td>495</td>
<td>400</td>
<td>35</td>
<td>236</td>
</tr>
<tr>
<td>1.45</td>
<td>1.45</td>
<td>4000</td>
<td>550</td>
<td>2000</td>
<td>275</td>
<td>236</td>
<td>35.6</td>
</tr>
</tbody>
</table>

where the incremental resistance change due to \( \Delta \delta \) cycles is

\[
\frac{\Delta R}{R_1} = \left( \frac{\Delta R}{R_1} \right)_{1+2} - \left( \frac{\Delta R}{R_1} \right)_{1} \tag{18}
\]

The value of \( (\Delta R/R_1)_{1+2} \) is the resistance change associated with the damage \( d \) defined in eq (14) and

\[
(\Delta R/R_1)_{1+2} = (\Delta R/R_1)_{CR} \quad \text{when} \quad d = 1 \tag{19}
\]

Although this discussion deals with a two-level load, eqs (17) and (19) can be extended to include \( k \) levels with

\[
(\Delta R/R_1)_{1+2+...+k} = (\Delta R/R_1)_{CR} \log^{-1} \left\{ A(\Delta \delta) + B(\Delta \delta) \log \left[ \sum_{i=1}^{k} (n_{in} + n_i) \right] \right\} \tag{20}
\]

A digital computer can be programmed to obtain numerical results for eq (20) and to tabulate \( (\Delta R/R_1)_{CR} \) together the damage \( d \) whenever the value of \( k \) is large.

To illustrate this analytical approach, consider an aluminum alloy 6061-T6 with the fatigue properties described in Fig. 9. It should be noted that the \( \Delta \varepsilon \) - \( N_f \) curve representing the 6061-T6 alloy closely matches the output of the fatigue-damage indicator with \( (\Delta R/R_1)_{CR} = 0.36 \) for \( 5 \times 10^5 < N_f < 10^6 \).

Five different loading conditions were treated where a fatigue-damage indicator was cycled until the damage \( d = 1 \) for this aluminum alloy. The results obtained for the five different loadings are shown in Table 1 and they indicate that \( (\Delta R/R_1)_{CR} \) is a constant equal to about 0.36.

It is also possible to establish the critical response of the fatigue-damage indicator by a graphical procedure which is illustrated in Fig. 10. In this example, the first block of cycles was at \( \Delta \varepsilon = 0.47 \) percent, \( N_f = 40,000 \), \( n_1 = 10,000 \) with \( d_1 = 0.25 \). The response of the gage is given by point A of Fig. 10 with \( (\Delta R/R_1) = 0.31 \). The second block of cycles was at \( \Delta \varepsilon = 0.27 \) percent, \( N_f = 10^6 \), \( n_2 = 7.5 \times 10^5 \), \( d_2 = 0.75 \).

The equivalent number of cycles at the second strain level is established by the intersection of the \( (\Delta R/R_1)_{CR} = 0.31 \) curve with the second strain level thus locating point B. The equivalent number of cycles at point B is \( 2.8 \times 10^3 \). The second block of \( 7.5 \times 10^5 \)

Fig. 10—Graphical determination of \( (\Delta R/R_1)_{CR} \) for a two-level fatigue loading

Fig. 9—Matching the fatigue characteristics of Al 6061-T6 to the response of a damage indicator. \( \sigma_u = \sigma_m \)
cycles at $\Delta e_2$ produces the response indicated by tracking from point B to point C. At point C, the total damage $d = 1$ and the value of $(\Delta R/R_1)_{CR}$ is constant regardless of the number of cycles and the strain range required to produce failure.

For more complex fatigue loadings where multi-level loads occur, the indicator also appears to predict failure with a critical value of $R/R_1$ which is constant for a specified material. This conclusion assumes a perfect match of the indicator to the material and also that the material follows Miner's damage law.

Finally, it should be clearly indicated that while the fatigue indicator employing composition D does closely match several materials, this particular composition is in no sense universal. Other compositions with different particle sizes and weight fractions and with different matrix constituents will respond differently to fatigue exposures. Thus, with considerably more research to classify families of compositions, it should prove possible to specify the most suitable composition for each material to predict the onset of fatigue failure from a simple resistance determination.

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References