An Engineering Approach to Shot Peening Mechanics

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ABSTRACT

A simple theoretical analysis of the shot peening process is presented. The process is viewed as one involving multiple and progressively repeated impact. The residual stress field under each impact interacts with similarly produced neighbouring fields to finally produce a residual stress distribution varying in depth but uniform in planes, parallel to the surface. Non-dimensional numbers which are useful for similarity considerations are introduced. New concepts such as shakedown, reversed plastic yielding and Bauschinger effect as well as strain rate are thought to contribute a significant role in the process.

KEYWORDS

Analysis, Residual stress distribution, Arc height, Shakedown, Nondimensional numbers.

INTRODUCTION

It has been apparent for some time that there is a huge demand for controlling the subsurface stress distribution in peened components. This has been confirmed by almost all the sessions at the first International Conference on Shot Peening held in Paris (1). The key to the role which the shot peening process plays in fatigue life, surface finish, stress corrosion and corrosion fatigue is in the manner in which shots interact with the target surface to engender post impact residual stresses as well as metallurgical changes.

It is firmly believed that any progress made in understanding the 'micro' and 'macro' deformation mechanisms will not only lead to improved design but also to expedient expansion of the range of applications of the process.

To date, there has been an extensive amount of experimental work on Almen intensity, arc height development, residual stress measurements, fatigue life, etc. Notwithstanding, there is a noticeable lack in the literature of analytical work on shot peening. This is not surprising, however, due to the complex manner in which target materials respond to
the multiple impact of shots. It is cumbersome to attempt to predict the impact response of a material to a single blow, as this would require fundamental understanding of high strain rate elastic as well as inelastic phenomena. The requisite understanding of single blow impact is still incomplete, but considerable progress has been achieved in recent years. The present level of comprehension is reviewed in the books by Goldsmith (2) and Johnson (3). This knowledge of high strain rate phenomena can be used to develop a physical description of the sequence of events that accompany the impact of a single projectile. When multiple impact takes place, the problem becomes more complex as it will require understanding of other aspects such as shakedown, progressive plastic deformation, Bauschinger effect and strain softening due to cyclic plastic deformation. Furthermore, the contact characteristics between thick target and projectiles and the concomitant stress waves will need to be examined.

Over the last few years, few reasonable advances have been made at UMIST towards answering some of these questions. The mechanical aspects of residual stress development which pertain to the influence of shot size, material properties and velocity as well as target strength characteristics on depth of the compressed layer, may be found in Refs. (4), (5) and (6).

Further development of the theoretical models as well as carefully constructed experiments are necessary before serious advances can be gained in understanding concomitant processes such as shakedown and reversed plastic yielding during peening.

This paper aims to shed further light on some of the important aspects of this process and to present a summary of a simplified analysis of the mechanics of the process which relies upon major engineering assumptions.

IMPACT GENERATED PLASTIC ZONE

The problem of the response of an elastic-plastic semi-infinite solid to a rigid sphere impact is one in which a number of difficulties exist. Apart from aspects of dynamic contact, there are still the processes of elastic and plastic stress waves due to a changing boundary condition unsolved. A single shot impinges obliquely and, thus, cratering of the target when being peened takes place under normal as well as tangential forces. It is beyond the scope of the present paper to address such aspects. However, one can gain a reasonable insight into the problem by considering the shot to be rigid, impinging upon a rigid perfectly plastic target. This oversimplification assists us in predicting the depth of the dent produced in the target surface and hence to later enable the estimation of the plastic zone \( h_p \). With the latter predicted, it is then possible to predict the residual stress distribution in a shot-peened strip of metal.

Let us consider the motion of the spherical shot as it normally impinges upon the surface. The equation of motion during contact is

\[
\frac{4\pi}{3} \rho R^3 \frac{dv}{dt} = -\pi a^2 \bar{p}
\]

where \( \rho \), \( R \) and \( v \) are the density, radius and velocity of the shot and \( a \) is the radius of the contact circle.

The average pressure \( \bar{p} \) resisting the motion has been shown to be given by, see ref. 5.
\[
\bar{p} = 0.6 + \frac{2}{3} \ln \frac{E_a}{YR}
\]  
(2)

where \(Y\) is the yield strength and \(E\) the Young's modulus of the target.

The appropriate non-dimensional "deformation parameter" is \(E_a/YR\) which effectively represents the ratio of the imposed strain \((a/R)\) to the capacity of the material to sustain elastic strain \((Y/E)\). The elastic limit is reached when putting \(\bar{p} = 1.07Y\) giving \(E_a/YR = 2\) and the fully plastic strain is reached when \(\bar{p} = 3Y\) giving \(E_a/YR = 36.6\). The latter values compare well with experimentally obtained values of 2.55 and 40 respectively. Of course Eq. (2) is not expected to give realistic predictions at values larger than 40, wherein a rigid plastic theory would be more appropriate in which \(\bar{p} = 3Y\).

Substituting Eq. (2) into (1) and integrating, using \(\frac{dv}{dt} = \frac{dv}{dz}\) we get

\[
\frac{Z}{R} = \left(\frac{2}{3}\right)^{\frac{1}{2}} \left(\frac{\nu}{\bar{p}}\right)^{\frac{1}{2}} \left(\frac{Z}{R}\right)_{0}^{n}
\]  
(3)

where \(\nu\) is the initial impact velocity and \(\bar{z}\) is the final indentation, with \(\bar{p} = 3Y\) and \(1/0 = [(a/2 + 2/9 \ln E/Y) + 1] [\ln(2Z/R) - 4Z/R]^{1/2}\). This equation governs the initial stages of deformation but as soon as the pressure reaches 3Y a rigid plastic analysis will hold. By assuming \(\bar{p}\) to remain constant during the indentation process, the solution of Eq. (1) gives

\[
\frac{Z}{R} = \left(\frac{2}{3}\right)^{\frac{1}{2}} \left(\frac{\nu}{\bar{p}}\right)^{\frac{1}{2}}
\]  
(4)

The non-dimensional number \((\nu \sqrt{2}/\bar{p})\) comes in most impact problems. It gives a measure of the severity of impact and is sometimes called the "Damage Number". It may be used to identify dynamic similitudes in shot-peening situations. It was suggested earlier, see refs. (4) and (5), that different peened targets may be considered to have the same arc height if they have the same Damage Number. This concept will be modified later in this text.

Results of experiments on Aluminium Alloy and steel target conducted at UMIST are included in Fig. 1, together with results from Clausen (7). The extensive set of impact results published by Clausen giving relations between \(Z\) and \(R\), \(v\) and hardness could all be reduced to one plot by use of the "Damage Number" concept. Even though, a strain rate index \(m = 0\) is used, i.e. a value for \(p = 3Y\) the quasi-static yield stress, the agreement between Eq. (4) and experiments is encouraging.

The dotted line represents a theoretical line of a modified version of eq. (4) allowing for strain rate with strain rate index \(m = 0.12\) in a stress-strain relation of the form:

\[
\sigma = \sigma_0 \left(\dot{\varepsilon}/\dot{\varepsilon}_0\right)^m
\]

where \(\sigma_0\) is the flow stress at a reference strain rate \(\dot{\varepsilon}_0\).

Fig. 1. Variation of indentation with the square root of Damage Number.
It may be seen that the results for steel are closer to the strain rate sensitive theory but those for the aluminium, being not as sensitive to strain rate, are closer to eq. (4) as would be expected.

The elastic-plastic stress field below each impact presents a difficult theoretical problem which is still a subject of serious research effort. The way ahead would seem to be via the finite element method. This is under current investigation by Al-Obaid (8). However, simplified approaches have been made by Al-Hassani (4) and (5) which gave reasonable agreement with experimental results. It is suggested that the relation

\[
\frac{h_p}{R} = \frac{\pi}{3} \left( \frac{e}{R} \right)^{1/3}
\]  

(5)

adequately holds for static and "dynamic" situations.

Thus, from eqns. (4) and (5), the impact generated plastic zone may be represented by

\[
\lambda = 3 \left( \frac{2}{3} \right)^{1/3} \frac{R}{h} \left( \frac{\rho_v}{P_0} \right)^{1/4}
\]  

(6)

where \(\lambda = h_p/h\) and \(h\) represents the thickness of the target. This equation shows that a modified non-dimensional number given by \([\rho_v^2 R^{1/4} / P h]\), is more suitable as a parameter for shot peening. It suggests that if this number remains the same, a geometrical similarity is achieved between the plastic layer and target thickness.

### RESIDUAL STRESSES ON SHOT REBOUND.

Let us consider the stress rate of a target element in the vicinity of the impacted surface. It was shown in ref. 5, that an element at the contact surface undergoes a cyclic tension-compression during each impact. As the point of maximum tension on the surface changes state of stress from yield in tension to plastic compression, a condition of reversed yield takes place. When this point comes under the shot the stress suddenly jumps to \(3Y\). If the material were to harden isotropically, the yield stress on unloading would exceed that during the loading by the same factor. But, for most materials the Bauschinger effect results in reversed yield at a stress less rather than greater than the initial loading yield.

If the element is on the vertical centre line within the plastic zone, the stresses on it are such that,

\[
\sigma_z - \sigma_r = -Y
\]  

(7)

where \(\sigma_z\) and \(\sigma_r\) are the axial and radial stresses (cylindrical coordinates). To represent unloading, we require to superpose the elastic stresses due to a uniform distribution of reversed pressure "tension" of magnitude \(3Y\) at the contact surface. The consequence of so doing is fully explained in ref. 5.

In shot peening, the far field stress due to one indenter adds to the stress below the other indenter if load application occurs simultaneously. Consequently, plasticity may be reached earlier and the plastically deformed depth may tend to be slightly shallower. However, in practice, the occurrence of impact is random and also includes multiple single as well as repeated impacts. The plastic zones below the shots join together to form an upper layer of residual compressive stress almost uniform over the active surface of the medium.
Impact On Prestressed Surface and Concept of Shakedown - Residual stresses introduced by plastic flow due to initial impact act in a manner to inhibit plastic deformation during subsequent impacts. Let us consider the target to have a uniform biaxial prestress \( \sigma_r = \sigma_0 \) introduced by the previous indentation process. For equilibrium on the free surface \( \sigma_z = 0 \). Now to maintain elastic state of stress we require

\[
(\sigma_r + \sigma_z) - \sigma_z \leq Y. \tag{8}
\]

For \( \sigma_r = -kY \) we have

\[
\sigma_r - \sigma_z \leq (1 + k)Y. \tag{9}
\]

Now for a sphere indenting a surface we have \( (\sigma_r - \sigma_z) \max = 2\tau \max = 0.936\bar{p} \) at \( z = 0.47a \),

For no yield we have

\[
\bar{p} \leq 1.07(1 + k)Y \tag{10}
\]

which is \( 1 + k \) times the mean pressure to just yield a stress free target.

Therefore, if the first impact were to introduce a biaxial residual stress of compression equal to the yield stress \( (k=1) \), the mean pressure required in the second impact, if yield is to occur should be doubled and the load be 8 folds.

If, however, the same impact pressure were to be applied, yield will not take place in subsequent loading cycles and the surface is said to Shakedown.

It is common in industry to apply a prestress on the specimen prior to peening. This process is called "Stress Peening". The direction of the imposed prestress is such that it enhances the effect of shot peening, i.e. to require less impact force to produce the same residual stress condition. Equation (10) shows that if prestress \( \sigma_r \) and \( \sigma_z \) were to be applied in the reverse direction, i.e. \( k < 0 \), the condition of no yield would require less average pressure. For \( k = -0.5 \), the required impact pressure would be halved and the impact load reduce to \( 1/3 \). The significance of \( k \) in determining the efficiency of peening is paramount. In real materials strain hardening and thermal softening adjust the value of the effective yield stress from one impact to the other and Eqs. (8) - (10) would have to take the concomitant values of \( Y \). Further discussion on dynamic shakedown of a strain hardening target is given in ref. 5. Enhancement of the condition of shakedown due to shots landing on pre-dented positions is also discussed in the same reference.

### Residual Stress Distribution

The overall residual stress in the shot peened target is the sum of all the fields caused by repeated impacts at each spot as well as progressive impacts to cover the whole surface of the target.

It is beyond any theoretical analysis to predict the instantaneous build up of the residual stress distribution. But, by making use of the measured patterns of residual stress in shot peened specimens one is encouraged to make a brave assumption that the rest of the target react to the total sum of the residual stress distribution by exhibiting an
equivalent direct and bending stresses acting in a manner to balance the internal stresses. Consequently, these cause bending and axial strains which are manifested by curvature of the target towards the impinging shots. This action continues until such time when the arc height levels off and a state of saturation is reached.

By envisaging that the bending moment and axial force are reacting to a causal stress or a "source stress", Al-Hassani (5) predicted a residual stress distribution which is valid at regions below the surface. An improved prediction was achieved by making use of an empirical relation originally proposed by Flanagan and Nikula (10) to fit experimentally measured residual stress in thick targets. This is a cosine function with its peak at \( z = ah_p \) and has a zero value at \( z = h_p \).

The residual stress distribution in a thin plate was considered to be the result of further addition of the bending and axial stress to this "source stress". Thus, the distribution is predicted to be

\[
\sigma(z) = \frac{E \varepsilon_m}{1-\nu} \left[ \frac{12\lambda}{\pi h} (1-\alpha) \left( \frac{h}{2} - z \right) C_1 + \frac{2\lambda}{\pi} (1-\alpha) C_2 \cos \pi \left( \frac{z-ah_p}{2(1-\alpha)h_p} \right) \right] \phi \tag{11}
\]

where \( \nu \) is the Poisson's ratio, \( \phi(h_p) \) is a unit step function equal to 1 for \( z \geq h_p \),

\[
C_1 = C_2 - 2\lambda + 4\lambda \left[ \frac{(1-\alpha)}{\pi} \cos \frac{\pi \alpha}{2(1-\alpha)} \right] \tag{12}
\]

and

\[
C_2 = 1 + \sin \frac{\pi \alpha}{2(1-\alpha)} \tag{13}
\]

The maximum strain \( \varepsilon_m \) is obtained from consideration of plane sections remain plane in a beam of length \( L \) bent into an arc height, \( \delta \). Thus,

\[
\varepsilon_m = \frac{\pi \rho \delta}{\lambda L^2 (1-\alpha) C_1} \tag{14}
\]

A set of predicted residual stress distribution results from eq. (11) are shown in Figs. 2 and 3. The conditions of peening for these results are for a shot of size \( R = 0.36 \text{ mm} \), density \( \rho = 7740 \text{ kg/m}^3 \) and target material with \( Y = 505 \text{ MN/m}^2 \), \( E = 200 \text{ GN/m}^2 \), \( \nu = 0.3 \) assumed to be a plate of \( L=76\text{mm} \). The value of \( \alpha \) is chosen to be 0.2. In Fig. 2 the thickness of target was assumed to be 0.76 mm and the curves are for shot impact velocities of 37, 73 and 91 m/sec respectively.

In Fig. 3 the influence of target thickness is demonstrated for an impact of 37 m/sec for \( h = 0.76 \text{ mm} \), 1.27 mm and for very thick target.

It is seen that the predicted curves agree with the common trends found in measured residual stress distributions. Other results may be obtained for glass beads and for targets of different materials.

**Arc Height and Coverage**

As the shots continue to bombard the surface, \( \delta \) increases up to saturation when full coverage and shakedown of all impacted spots is achieved.
To have an idea of the extent of surface stretching imposed by the shots we re-examine the single shot motion again. It may be shown that the volume of the crater formed in a rigid perfectly plastic target by a projectile of mass \( M \) impinging with a velocity \( v_o \) is,

\[
V = \frac{Mv_o^2}{2\rho} \tag{15}
\]

The change in surface area is due to indentation of the sphere to a depth \( z \) is from geometry \( \approx \pi z^2 \). The crater volume may be expressed by \( \pi a^2 R/4 \). Thus with \( a^2 = 2zR \) we get \( V = \pi z^2 R \Delta A \). Hence for each freshly impacted spot,

\[
\Delta A = \frac{Mv_o^2}{2\rho} \tag{16}
\]

If each spot is independently formed and with \( N \) shots per second covering the surface, the total increase in surface area of the peened surface would be

\[
\Sigma \Delta A = \left( \frac{Nv_o^2}{2\rho} \right) t \tag{17}
\]

One is tempted to dare to introduce a rudimentary analysis to explain some aspects of the arc height vs. time curve.

Let us assume that at time \( t \) the actual area covered by spots is \( C \) times the actual area under peening. The distribution of shots can be represented by a Poisson's statistical relation which simplifies to give

\[
\text{actual area covered} = \text{area} (1-e^{-Nt}) \tag{18}
\]

where \( Nt \) here represents the total number of coverage runs which for convenience will be assumed equal to total number of spots.

The average overall surface strain, which is responsible for arc height development is therefore,

\[
\varepsilon_s = \frac{NMv_o^2}{2\rho R L} t (1-e^{-Nt}) \tag{19}
\]

The arc height is related to \( \varepsilon_s \) through \( \delta = \varepsilon_s L/8h \). Hence (with \( M = \frac{4}{3} \pi \rho R^3 \))

\[
\frac{\delta}{h} = \frac{\pi}{12} \frac{R^2}{h^2} \left( \frac{\rho v_o^2}{c} \right) \frac{Nt (1-e^{-Nt})}{Nt (1-e^{-Nt})} \tag{20}
\]

This equation is by no means accurate as it ignores the shakedown in spots repeatedly impacted. This will manifest itself by higher values of \( \rho \) at increased \( Nt \). Experimental results confirm the tendency of eqn. (20).

It can now be seen that in order to maintain similarity in Almen intensity, the nondimensional parameters \( (R^2/h^2)(\rho v_o^2/\rho) \) must be maintained the same. Whilst similarity in plastic layer i.e. to keep \( h/\rho \) the same, the non-dimensional parameters \( (R^2/h^2)(\rho v_o^2/\rho) \) must be maintained the same.
CONCLUDING REMARKS

Shakedown, Bauschinger effect and strain rate effects seem to have an appreciable role in the mechanics of shot peening. A simplified model was successfully used to bring out the main features of the process. Residual stress distribution and arc height as well as thickness of plastic layer were predicted. Non-dimensional parameters for dynamic and geometric similarity are proposed.

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REFERENCES

7. Clausen, R., see ref. 1, pp.279-285.