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HELICAL COIL SPRING DESIGN

Coil springs are widely used throughout industry but they rarely receive the attention they deserve from designers. Here, Derek Saynor* explains the main factors influencing spring design

It is probably because they are so commonplace that helical coil springs are frequently regarded as low grade components by designers. As a consequence they are rarely given sufficient attention at the design stage, yet are invariably required to operate reliably for many years under arduous conditions and at stress levels far above other components within the mechanism.

To ensure a successful design it is essential that the spring be considered at an early stage in the overall design process. In recent years this has become possible through computer-aided design (CAD) of springs. The Spring Research and Manufacturers' Association (SRAMA) has been at the forefront of this development and has produced a package of design and checking programs for all types of coil springs.

Because of the program's speed the designer is able to consider the spring at the earliest possible stage and ensure sufficient space is made available for the spring taking due account of all the factors affecting the design.

Design criteria

The following factors influence the spring design and dictate the type of spring required, the material to be used and the spring's detailed dimensional and load characteristics:

- *Type of load application.* Whether the motion required is axial compression, axial extension or rotational. This will dictate whether a compression, tension or torsion spring should be used.
- *Environmental factors.* The extremes of temperature to which the spring will be exposed; any potentially corrosive environment; special requirements such as electrical conductivity or constant modulus with temperature.
- *Available space envelope.* The space available to accommodate the spring should be as large as physically possible to minimise stresses within the spring.
- *Loads and deflections required.* What maximum and minimum loads are required from the spring and at what positions? Also how accurate do the loads need to be for a successful operation of the mechanism?
- *Number of operating cycles.* What is the fatigue life required from the springs? The greater the number of cycles the lower will be the allowable operating stresses.
- *Life expectancy requirement.* The load-carrying capacity of springs reduces with age due to creep within the material while under load.

These criteria should be clearly established by the designer before commencing detailed design of the spring.

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Once a specification has been drawn up for the spring the design process can begin. The first step is to select the spring material.

There is a wide choice of materials available. Wherever possible a relevant British Standard material should be used. The full range of spring materials normally available can be subdivided into the following broad headings:

- carbon and low-alloy steels;
- stainless steels;
- heat-resisting steels and alloys; and,
- copper alloys.

Materials specifications

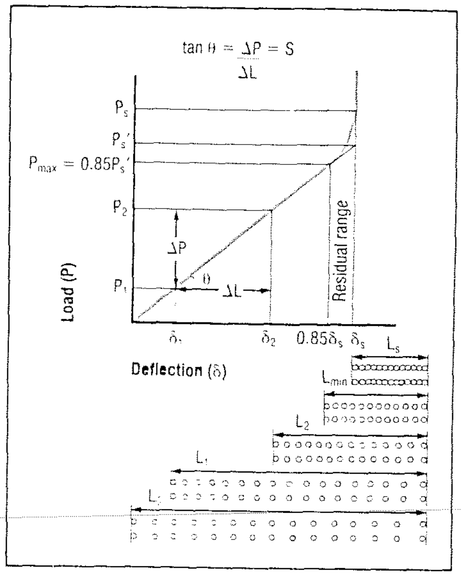
Carbon and low-alloy steels are the most common and least costly spring materials and there are four main types: patented cold-drawn steel wire to *BS 5216:1975;* prehardened and tempered carbon and low-alloy steel wire to *BS 2803;* annealed carbon and low-alloy steel wire to *BS 1429:1980;* and annealed carbon and low alloy steels to *BS 970:Pt 5:1970,* for hot formed springs.

Stainless steels are used when resistance to corrosion or relaxation is required. There are three main types used – austenitic, martensitic and precipitation hardening – all contained in *BS 2056:1983.*

The austenitic stainless steels are available in two grades: 302S26 in two tensile strength ranges, for most general applications, and 316S42, a molybdenum-bearing grade for sea-water and salt spray environments. The martensitic stainless steels are produced in one grade, 420S45, requiring hardening and tempering after spring coiling. Similarly, precipitation hardening stainless steel is also produced in a single grade; supplied cold drawn and polished but requiring an ageing heat treatment after spring coiling to develop full spring properties.

A broad range of proprietary nickel and cobalt alloys as well as certain grades of tool steel to *BS 4659* are used for elevated temperature applications. The nickel and cobalt alloys also provide exceptional corrosion resistance. The most commonly used grades are: Nimonic alloy 90, Inconel alloy 600, Inconel alloy X750 and Monel alloy K500.

Copper alloys do not permit high working stresses, but are non-magnetic, have high electrical conductivity and good corrosion resistances. Amongst those commonly used in spring manufacture are: brass to *BS 2786:1963;* phosphor-bronze and beryllium-copper. The last two materials are both to *BS 2873:1969.*



1 *The load/deflection curve for a helically coiled compression spring*

Compression springs are the most common type of coil spring as they are generally cheaper and easier to manufacture than tension or torsion springs. Compression springs should therefore be chosen in preference wherever possible.

The relationship between load and deflection for a compression spring is shown in Fig **1.**

As the spring is compressed the load increases linearly at a rate known as the spring rate or stiffness *(S).* Ideally this continues until the solid length (L_s) is reached at which the load would be P_s' (known as the theoretical solid load). No further deflection is possible from this point. In practice, however, the line departs markedly from linearity near solid so that the actual solid load becomes P_s. This non-linear zone is known as the residual range and must be avoided in service for reasons discussed later.

For compression springs the spring rate is given by:

$$S = \frac{Gd^4}{8nD^3} = \frac{Gd}{8nc^3}$$

where G = modulus of rigidity (N/mm²)
d = wire diameter (mm)
n = number of active coils
D = mean coil diameter (mm)
c = spring index ($c = D/d$)

Once the spring rate is known, the load *(P)* at any position *(L)* can easily be determined from:

$$P = S(L_0 - L)$$

where L_0 = free (unloaded) length of spring (mm)

The modulus of rigidity (or torsional modulus) is dependent on the material used. Values of G at room temperature are given in Table **1.**

The spring index *(c)* indicates how tightly the spring is wound. This value should not lie outside the range 3.5 to 20 (preferably 4 to 10) and must never be less than 2.5. Active coils are those which deflect under load. Their number cannot be directly measured by experiment. Mathematical relationships exist, however, which enable the number of active coils to be estimated. Their number depends upon the type of spring end used, as shown in Table **3**. These values, however, are not exact due to variations in end coil seating and closure.

Five types

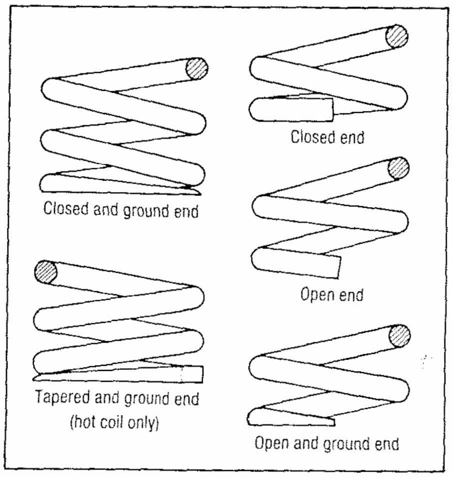
The form of ends affects both the number of active coils and solid length, so it must be selected at an early stage. Five types are possible (see Fig **2**). Closed and ground are the most common providing a highly stable seat with good load axiality. Where these features are less important than unit cost, and on fine wires less than 1 mm dia where grinding presents problems, closed ends are frequently used.

Both open ends and open-and-ground ends are rarely used except when solid length is severely constricted. The springs are highly unstable and require expensive seats with a special helical shape to match the spring. Hot coiled springs with large bar diameters have the bar ends tapered by hot forging prior to coiling to minimise the grinding necessary. Such ends are termed tapered (or forged) and ground.

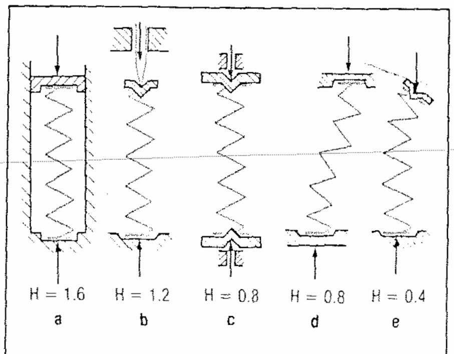
Springs should not normally be compressed solid in service as normal load tolerances cannot be maintained close to the solid length and coils remaining active near solid carry disproportionately high loads and stresses. Furthermore fretting and surface damage can occur causing premature failure. To avoid such

Materials	Modulus of rigidity (G), kN/mm²	Youngs Modulus (E), kN/mm²
Hard-drawn carbon steel	79.3	206.8
Carbon steel	79.3	206.8
Silicon-manganese steel	79.3	206.8
Chromium-vanadium steel	79.3	206.8
Martensitic stainless steel	79.3	206.8
Precipitation hardened stainless steel	75.8	203.4
Austenitic stainless steel	70.3	187.5
Phosphor-bronze	41.0	103.0
Hard-drawn brass	36.0	103.0
Beryllium-copper	48.3	127.6
Monel alloy K-500	65.3	178.7
Inconel alloy 600	75.6	206.2
Nimonic alloy 90	82.5	213.0-240.0
Inconel X750	75.6	213.0

Table 1: Values of G and E for common spring materials at room temperature



2 *End coil forms for compression springs*



3 *End fixation factor (H) for springs with:* **a**, *both ends fixed laterally guided;* **b**, *one end pivoted laterally guided;* **c**, *both ends pivoted laterally guided;* **d**, *both ends fixed laterally unguided; and,* **e**, *both ends fixed laterally unguided*

problems the maximum service load should be limited to 85% of the theoretical solid load, leaving a 15% residual range (*ie* $P_{max} = 0.85P_s$).

Depending upon the method of end fixing and the slenderness ratio (L_0/D) a compression spring may exhibit lateral instability and buckle at a critical deflection. Buckling may be predicted using the end fixing constants from Figs **3** and **4.**

The torsional stress in a compression spring is given by

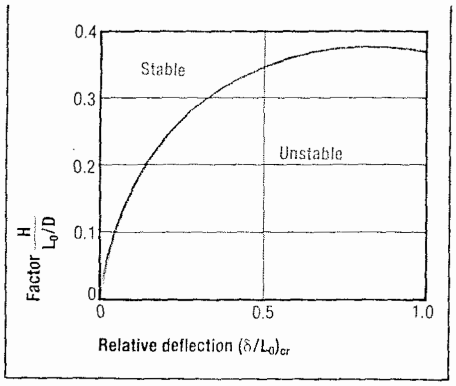
$$\tau = \frac{8PDK}{\pi d^3}$$

where K = curvature correction factor

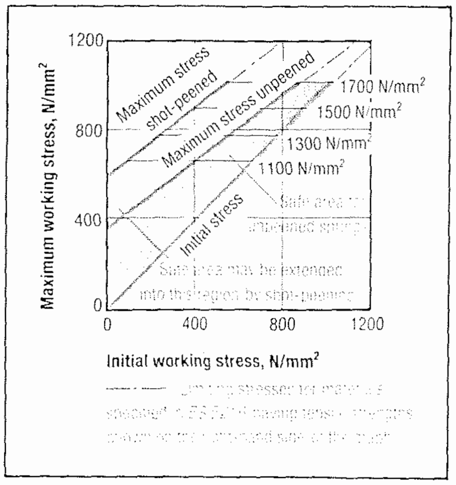
$$K = \frac{c+0.2}{c-1}$$

The maximum stress is the stress at solid. The solid stress must be limited to the values given in Table **2** related to the UTS of the material.

Pre-stressing is an additional manufacturing operation which allows the use of higher solid stresses in springs by raising the apparent elastic limit in torsion. The process also has a beneficial effect on fatigue life. Other stress conditions which must be considered by the



4 *The critical relative deflection for buckling*



5 *The Goodman fatigue diagram for BS 5216 compression springs*

designer are those which occur under fatigue and relaxation conditions. If a spring is required to have a life in excess of 10 000 cycles then the maximum and minimum operating stresses must be calculated and reference made to fatigue data for the relevant grade of material on a Goodman diagram (see Fig **5**). To be confident of survival for the particular number of cycles shown on the diagram the combination of maximum and minimum stress must lie within the safe area.

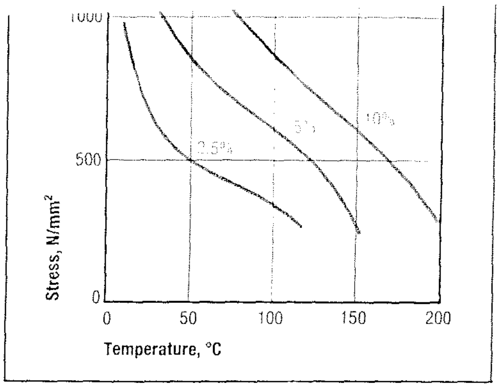
Shot-peening

It can be seen that shot-peening extends the safe zone considerably, indicating that shot-peened springs can operate successfully under more arduous combinations of stress than unpeened springs.

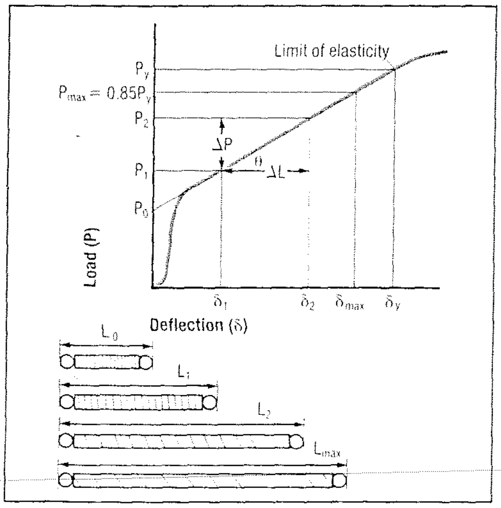
Relaxation is a loss in spring load with time under the influence of stress usually combined with elevated temperature. It is a manifestation of creep and must be considered if the spring is to be compressed for long periods, particularly at high temperature.

The effects of time, temperature and stress on relaxation are shown in Fig **6.** If the level of relaxation is unacceptable then it will be necessary to reduce operating stresses by re-design or changing the material to a more-relaxation-resistant alloy. Alternatively a process known as hot pre-stressing can be carried out, in which the spring is pre-stressed for a short time at high temperature.

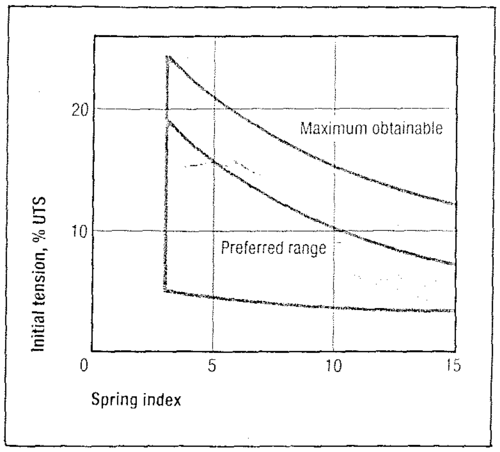
Springs cannot be manufactured to the same level of tolerances as machined components. They are also more difficult to measure repeatably using standard measuring techniques. Typical commercial tolerances for springs are



6 *Iso-relaxation curves for patented carbon steels after 72h*



7 *The load/deflection curve for a tension spring*



8 *Maximum initial tension versus spring index*

Material	Specification	Maximum working and solid stresses: Prestressed compression springs Working % UTS	Solid % UTS	Unprestressed compression and stress relieved Working % UTS	tension springs Solid % UTS	Unprestressed torsion springs Working % Rm
Patented cold drawn spring steel wire	BS 5216	60	70	42	49	70
Prehardened and tempered carbon steel wire	BS 2803	60	70	45	53	70
Austenitic stainless steel wire	BS 2056	50	59	35	40	70
Martensitic stainless steel wire	BS 2056	60	70	45	53	70
Precipitation hardening stainless wire	BS 2056	60	70	45	53	70
Prehardened and tempered low alloy steel wire	BS 2803	60	70	45	53	70
Steels hardened and tempered after coiling. Carbon and low alloy	BS 1429 and BS 970	60	70	45	53	70
Hard drawn brass	BS 2786	50	59	35	41	70
Extra hard phosphor-bronze wire	BS 2873	50	59	35	40	70
Beryllium-copper wire	BS 2873	50	59	35	40	70
Monel alloy K 500		45	53	34	40	70
Nimonic alloy 90		47	55	36	42	70
Inconel alloy X750		47	55	34	40	70
Inconel alloy 600		47	55	36	42	70

Table 2 Maximum permissible working stresses for various materials

specified in *BS 1726*.

The relationship between load and deflection for a tension spring is shown in Fig **7.** As the load is applied to the spring no extension takes place until a load known as the initial tension load (P_0) is exceeded. This is a load which holds the coils in contact and is wound into the spring during coiling. In practice some small overall deflection takes place before P_0 is exceeded since the hooks on the spring end open out before the coils begin to separate. Once P_0 is exceeded the spring extends linearly at a rate *(S)* up to the yielding load (P_y) corresponding to the elastic limit of the material in torsion. The maximum service load (P_{max}) must be limited to 85% of the yield load to ensure overstressing is avoided in service and during installation.

For tension springs the rate is given by the same formula as compression springs,

$$\text{ie } S = \frac{Gd^4}{8nD^3}$$

The load *(P)* at any length *(L)* is given by:

$$P = P_0 + S(L - L_0)$$

The maximum level of initial tension which can be produced depends upon the UTS of the material and the spring index (see Fig **8**). Stress relieving after coiling reduces the 'as coiled' initial tension but allows higher maximum operating stress levels as shown in Table **2.** Both high and low levels of initial tension are difficult to coil consistently hence values in the middle range are recommended.

All coils are active in tension springs. Consequently, the partial number of coils is affected by the relative angular orientation of the end hooks (except where hooks are of the swivel or insert type). Similar limitations on spring index exist for tension springs as discussed earlier for compression springs.

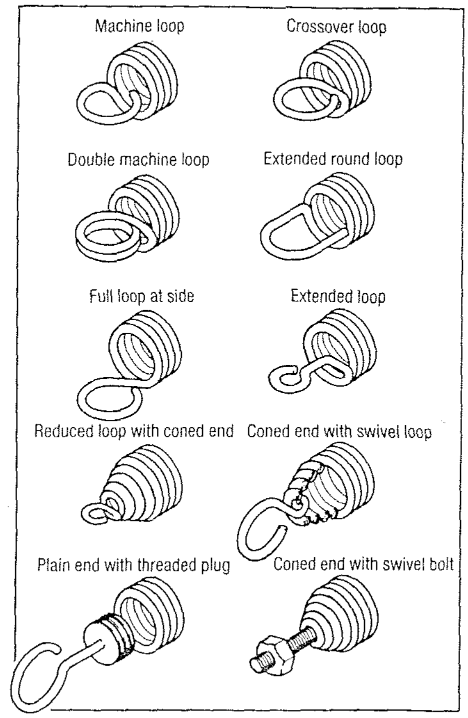
There is a vast range of hook forms possible on tension springs. Some common forms are shown in Fig **9.** The final selection is mainly influenced by the type of anchoring point on the component to which the spring must be attached. The minimum radii in bends must be limited to 1.5 times the wire diameter to avoid excessive stress concentration effects. In addition reduced ends are recommended to reduce bending stresses within the end hook.

The torsional stress in the body of a tension spring is given by:

$$\tau = \frac{8PDK}{\pi d^3}$$

The maximum allowable service stress (τ_{max}) corresponding to the maximum service load (P_{max}) is given in Table **2.**

Tension springs are less effective under fatigue conditions than compression springs. Typically, operating stresses must be reduced by 20 to 25% for similar lives. In general, tension



9 *Common hook types for tension springs*

springs fail at the position of maximum bending stress in the end hook hence the overall shape of the hook has a considerable influence. A further difficulty is that tension springs cannot be effectively shot-peened or pre-stressed to improve fatigue performance.

Frictional effects

The relationship between torque and rotation for a torsion spring is shown in Fig **10.** In theory, as torque is applied and released, the spring winds up and down in a linear manner along line *OA*. In fact, because of frictional effects, the torque increases along *OB* as the spring is wound up and decreases first along *BC* as the friction is released and then along *CO* as the spring is unwound.

The effect of friction is to produce the hysteresis loop *OBC* which produces substantial uncertainty in the torque at any angular rotation. The friction arises at three points within the system due to:

- relative movement of legs at the driving point;
- intercoil friction – torsion springs are usually close coiled to minimise overall axial length; and,
- mandrel friction – torsion springs are often supported on a central mandrel during operation and the applied forces are then opposed by reaction forces between the spring body and the mandrel.

In addition, the maximum available movement is reduced from θ' to θ_{max} (see Fig **10**).

Due to the uncertainties introduced by these frictional effects torsion springs should be used only in relatively low grade applications where torque requirements are not exact. They should never be operated against the direction of winding due to the extremely adverse stress pattern generated which severely limits their operating range.

For torsion springs (ignoring friction) the rate (S_θ) is given by:

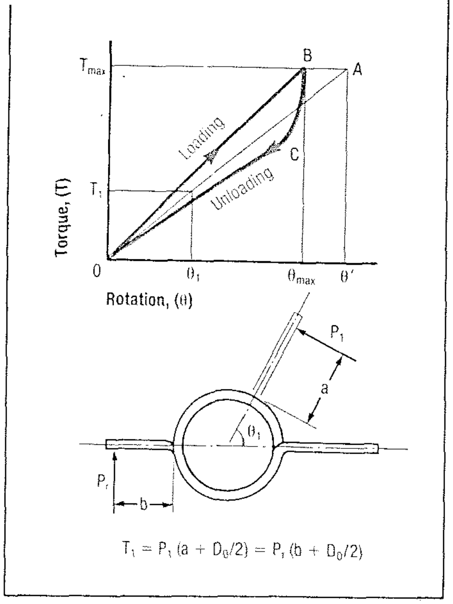
$$S_\theta = \frac{Ed^4}{3667nD} \text{(per degree)}$$

where n = number of coils
D = mean coil diameter (mm)
E = Young's Modulus (N/mm²)
d = wire diameter (mm)

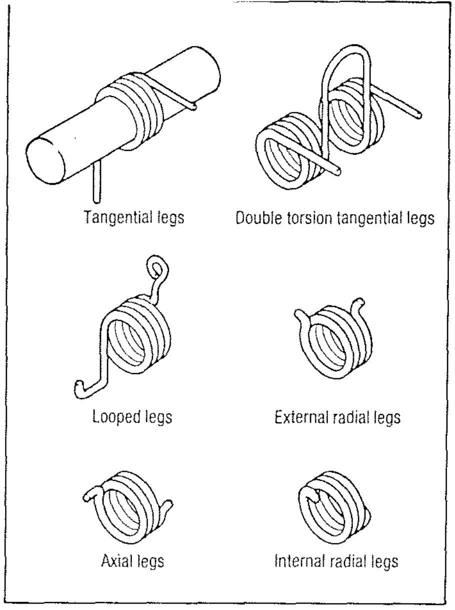
Type of End	Number of active coils (n)	Solid Length (L_s)
Closed and ground	$N-2$	$(N-\frac{1}{2})\times d$
Closed	$N-2$	$(N+1)\times d$
Open	N	$(N+1)\times d$
Open and ground	$N-1$	$N\times d$
Tapered and ground	$N-1.5$	$(N-\frac{1}{2})\times d$

d = wire diameter; N = no of total coils

Table 3 – Relationships between n, N and L, for compression springs



10 *Torque/rotation for a typical torsion spring*



11 *Some commonly encountered leg forms for torsion springs*

Due to the uncertainty caused by frictional effects, this formula is usually considered sufficiently accurate for most purposes. However, if the spring has only a few coils and the legs are relatively long, then it may be necessary to consider the contribution to the overall rate provided by flexing of the legs. This is taken into account using the following modified formula:

$$S_\theta = \frac{Ed^4}{1167(nD + 0.33\,(a+b))} \text{ (per degree)}$$

where a = length of first leg from point of load application (mm)
b = length of second leg from point of load application (mm)

Whichever formula is used the calculated rate can then be used to predict the torque (T) at any angular rotation (θ) using the formula $T = S_\theta\theta$.

Values of Youngs Modulus (E) for common spring materials are given in Table **1**. Similar limitations on spring index exist for torsion springs as for compression springs.

As a torsion spring is subject to an applied torque the consequent angular rotation of the spring causes the number of coils in the spring body to change. This change is offset by a similar change to the coil diameter which results in no change to the spring rate. However, these dimensional changes need to be considered when selecting any supporting mandrel size or when assigning available axial space. The reduced inside diameter D_i' when the spring is wound up is given by:

$$D_i' = \frac{360n}{360\,n + \theta}\,(D_i + d) - d$$

where n = number of turns in unloaded position
θ = angle of rotation (degree)
D_i' = inside coil diameter in unloaded position (mm)

The body length (L_b) of an unloaded close coiled torsion spring is given by $L_b = (n + 1)\,d$.

The increased body length L_b' when the spring is wound up is given by:

$$L_b' = (n + 1 + \theta/360)\,d$$

The design of the spring end is made primarily for the purpose of transmitting external torque to the spring. As a result torsion springs are made with a variety of shapes. See Fig **11**.

All coils are active in torsion springs (the legs also as discussed earlier). Consequently the partial number of coils is affected by the relative angular orientation of the legs. The minimum radii in bends must be limited to 1.5 times the wire diameter to avoid excessive stress concentration.

The bending stress (σ) in the body of a torsion spring is given by

$$\sigma = \frac{32T}{\pi d^3}\cdot K_b$$

where K_b = stress concentration factor in bending and

$$K_b = \frac{c}{c - 0.75}$$

The maximum allowable service stress (σ_{max}) corresponding to the maximum service torque (T_{max}) for stress relieved springs is given in Table **2,** as a percentage of the UTS.

Although torsion springs can be pre-stressed, to increase the allowable stress further, it is not recommended due to the prohibitive cost.

The fatigue performance of torsion springs is very variable due to frictional effects which cause surface damage due to fretting and increase the operating stress range. Shot peening is, however, beneficial since the position of maximum stress is on the outside of the coil and is exposed to the shot stream. Even so, the designer is strongly advised to avoid the use of torsion springs in fatigue applications.

In this article the basic design formulae and methods for the three most common coil spring types have been presented. The factors to be considered in material selection have also been highlighted. However, it must be admitted that the manual design of springs is an involved task; yet must be carried out rapidly if the design is to be considered at a sufficiently early stage to ensure a successful overall design. It is here that CAD techniques come to the fore ☐