THE EFFECT OF MATERIAL BEHAVIOUR LAW ON THE THEORETICAL SHOT PEENING RESULTS

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Abstract - In order to predict residual stress distribution profiles introduced by shot peening of metallic parts a model has been developed where major controlling factors are accounted for. This model allows to separate the contribution of shot peening parameters i.e shot type, size and velocity from the influence of the treated material i.e its constitutive law.

For materials characterised by a definite yield stress such as plain carbon steels and titanium alloys a simple approach based on elastic-plastic behaviour associated with linear kinematic hardening has revealed a fairly good approximation. Nevertheless for some materials of this kind theoretical results can be greatly improved if isotropic hardening is taken into account as devised by GUECHICHI et al [1986]. For plain carbon steels and titanium alloys experimental values of residual stresses as measured by using an X-ray diffraction technique fall in good correlation with the theoretically predicted profile.

On the other hand when applied to materials showing no definite yield stress such as aluminium alloys, nickel alloys and stainless steels theoretical prediction is not in good agreement with experimental evaluation. Hence we think a better description of the shot peened material behaviour is needed.

SHORT TITLE: Shot peening prediction.

I. INTRODUCTION

To improve fatigue life of metallic components especially in car manufacturing and aerospace industry shot peening is widely used. This cold working treatment consists in subjecting the surface to the impact of 'spherical' shots made of steel, ceramic or glass. A local plastic zone is generated by the impact of the shot on the treated surface, this creates compressive residual stresses which are thought to prevent initiation or propagation of cracks.

When exposed to a shots jet the material response to multiple impacts is a complex function of the characteristics of shots, the material properties and the processing conditions. Our working assumptions are the following:

- the shots are spherical,
- the impact of the shots is perpendicular to the treated surface,
- the shot speed is constant,
- the material loading produced by shot peening is a pseudo-cyclic one,
- the state of the material under the loading is a stabilised one,
- the treated material behaviour is elastic-plastic.

II. THEORETICAL MODEL

II.1. SHOT PEENING GLOBAL LOADING

The shot peened part is considered as a semi-infinite body which has been uniformly loaded so that a homogeneous residual stress field and associated plastic strain exist at any specified depth. The loading introduced by shot peening can be characterised on each fundamental volume element by the elastic response of the structure. A radial loading is assumed to give an accurate enough description of the real loading. Maximum purely elastic stress under the shot $\underline{\Sigma}_{max}$ and the nul stress field during unloaded state $\underline{\Sigma}_{min}$ will be sufficient to describe the purely elastic stress evolution :

$$\underline{\Sigma}^{el}(t) = \Lambda(t) * \underline{\Sigma}_{max} \qquad (1)$$

the scalar function of time $\Lambda(t)$ is equal to unity during the shot contact time and equal to zero after the shot has rebounded away from the surface.

II.2. PROJECTILE

The resulting residual stress distribution depends on projectile characteristics such as its size and nature. In fact the elastic-plastic energy transmitted by a sphere striking normally a semi-infinite body is :

$$W_{ep} = \frac{1}{12} (K * \pi * \rho * D^3 * V^2) \qquad (2)$$

where K is an efficiency coefficient which stands for thermal and elastic dissipation during the impact. According to JOHNSON [1972] the value of K is fixed at 80%. A 100% K value will describe purely elastic rebound energy. V is the impact velocity of the projectile, and ρ and D are respectively the material density and the diameter of the shot.

This hypothesis has been used by Davies [1949] to obtain the dynamic boundary conditions between the shot and the affected material.

II.3. FUNDAMENTAL EQUATIONS FOR THE ELASTIC LOADING

To describe the maximum elastic loading we have considered the impact of an elastic sphere on the surface of an elastic semi-infinite body as a particular application of Hertz's theory of contact between two elastic spheres.

Any point position in the affected layer is specified in terms of three cylindrical polar coordinates r, θ, z using the system (0, r, θ, z) the origin of which is located at the center of the indentation zone. (0, r, θ) is the plane boundary of the semi-infinite body and positive z-axis is directed from the surface inwards.

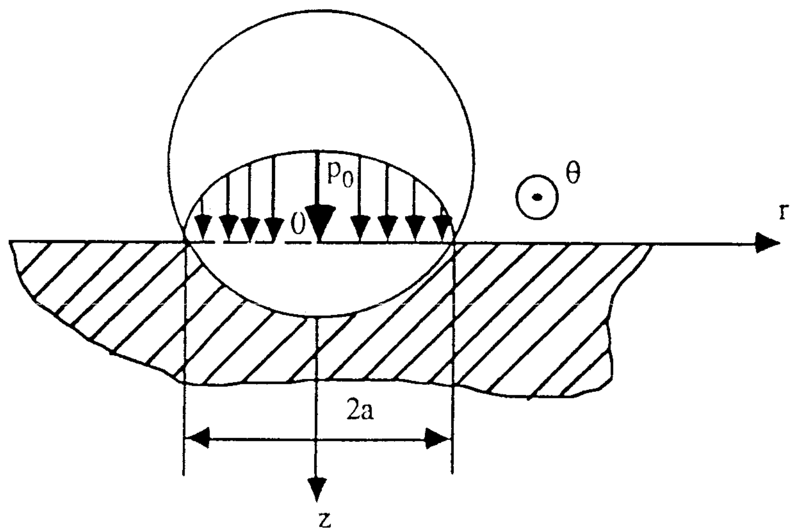


Figure 1. Elastic contact indentation.

when compression is at maximum the radius of the contact circle is expressed as:

$$a = \frac{D}{2} * \left[\frac{5}{2} * \pi * K * \rho * \frac{V^2}{E} \right]^{\frac{1}{5}} \qquad (3)$$

and derived maximum normal pressure is given by:

$$p_o = \frac{1}{\pi} * \left[\frac{5}{2} * \pi * \rho * V^2 * K * E^4 \right]^{\frac{1}{5}} \qquad (4)$$

Those two equations have been previously proposed by DAVIES [1949] where:

$$\frac{1}{E} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \qquad (5)$$

with :

E_1, ν_1 Young modulus and Poisson ratio of shot material,

E_2, ν_2 Young modulus and Poisson ratio of shot peened material.

Hertz results are used to modelise the elastic stress field created by the impact and as presented by SARDAIN [1981] the Hertzian elastic stress tensor can be written as follows :

$$\underline{\Sigma} = \begin{pmatrix} \Sigma_{rr} & 0 & 0 \\ 0 & \Sigma_{\theta\theta} & 0 \\ 0 & 0 & \Sigma_{zz} \end{pmatrix}.$$

the stresses reach their maximum under the shot and can be written :

$$\Sigma_{rr} = \Sigma_{\theta\theta} = p_0 (1 + \nu) [\frac{z}{a} \arctan (\frac{a}{z}) - 1] + p_0 \frac{a^2}{2 (a^2 + z^2)} \qquad (6)$$

$$\Sigma_{zz} = - p_0 \frac{a^2}{(a^2 + z^2)} \qquad (7)$$

II.4. FUNDAMENTAL EQUATIONS FOR RESIDUAL STRESS TENSOR

After a sufficient number of impacts, the residual stress tensor in the stabilised state is independent of the coordinates (r,θ) and remains constant on any plane parallel to the surface with $R_{rr} = R_{\theta\theta}$. The boundary conditions on the surface enable us to write:

$$R_{rz}(0) = R_{\theta z}(0) = R_{zz}(0) = 0 \qquad (8)$$

The equilibrium equations are reduced to:

$$\frac{\partial R_{zz}}{\partial z} = 0 . \tag{9}$$

So at any depth z we have:

$$R_{r\theta}(z) = R_{rz}(z) = R_{\theta z}(z) = R_{zz}(z) = 0. \tag{10}$$

The equilibrium equations and the boundary conditions for the residual stresses lead to the following residual stress tensor expression:

$$\underline{R} = \begin{pmatrix} R(z) & 0 & 0 \\ & R(z) & 0 \\ & & 0 \end{pmatrix}.$$

II.5. FUNDAMENTAL EQUATIONS FOR RESIDUAL STRAIN TENSOR

The half-space body is deformed in z direction only when treated by shot peening, so the kinematically admissible inelastic strain tensor, associated with residual stresses, is:

$$\underline{E}^{ine} = \begin{pmatrix} 0 & 0 & 0 \\ & 0 & 0 \\ & & E^{ine}(z) \end{pmatrix}.$$

Two components contribute to the inelastic strain, an elastic strain related to the residual stresses by means of the laws of elasticity and a plastic strain:

$$\underline{E}^{ine} = \underline{M}\ \underline{R} + \underline{E}^{p}. \tag{11}$$

As a result and taking into account the plastic incompressibility the plastic strain tensor is therefore:

$$\underline{E}^p \begin{pmatrix} -\frac{1}{2} E^p(z) & 0 & 0 \\ & -\frac{1}{2} E^p(z) & 0 \\ & & E^p(z) \end{pmatrix}$$

II.6. SUBJECTED MATERIAL

With the Hill normality flow rule we have used the Von Mises criterion written in the deviatoric space as:

$$\frac{1}{2} (S_{ij} : S_{ij}) - \tau_o^2 \leq 0 \qquad (12)$$

where S_{ij} is the stress deviator. However the elastic domain is represented in the deviator space by a sphere of radius R_o such as:

$$R_o = \sqrt{\frac{2}{3}} * \sigma_s = \sqrt{2} * \tau_o \qquad (13)$$

where σ_s and τ_o are respectively the semi-infinite body material yield stresses as measured in a uniaxial tensile test and shear test. The residual stress field is influenced by the specification (or the determination) of the yield stress, this is an essential point of the model. Because of the cyclic aspect of the shot peening load we use the cyclic yield stress.

II.7. BASIC MODEL

A simple and straightforward method is basically used to build up the residual stress prediction model. The residual stresses and the inelastic strain fields created by a cyclic loading are evaluated using the simplified method proposed by (ZARKA & CASIER [1972] and INGLEBERT <u>et al</u> [1985] [1989]).

II.7.1 LOCAL STUDY

The material volume element is considered as a microstructure composed of elementary perfectly plastic mechanisms. The local plastic strains $\underline{\alpha}$ of those micro-mechanisms are taken as internal variables. The generated local stresses on the micro-mechanisms are:

$$\underline{\sigma} = \underline{A}\ \underline{\Sigma}^{el} - (\underline{y} - \underline{A}\,R) = \underline{A}\ \underline{\Sigma}^{el} - \underline{Y} \qquad (14)$$

where $\underline{A}$ is the elastic localisation tensor in the volume element defined by MANDEL [1978], $\underline{y}$ is the material transformed parameters tensor (opposit to the residual stresses in the elementary mechanism), they are related to the internal variables by means of the coupling matrix $\underline{B}$:

$$\underline{y} = \underline{B}\ \underline{\alpha}. \qquad (15)$$

The plastic criterion is expressed as a function of $\underline{\Sigma}^{el}$, easily determined, and a local variable $\underline{Y}$ (structural transformed parameters) which includes the coupling effect due to the residual stresses $\underline{R}$.

The volume element global plastic strain created by the applied global stress $\underline{\Sigma}^{el}$ is:

$$E^{P} = \underline{A}^{T}\ \underline{\alpha} = \underline{A}^{T}\ \underline{B}^{-1}\ \underline{y}\,. \qquad (16)$$

More details about the material local study are given by INGLEBERT [1989].
The Von Mises criterion is written at the level of the micro-mechanism:

$$\frac{1}{2}(\sigma^{T} : \sigma) - \tau_{o}^{2} \leq 0\,. \qquad (17)$$

II.7.2. YIELD SURFACE EVOLUTION

In the transformed parameters space the yield surface evolution is perfectly determined if the variation of the global load $\underline{\Sigma}^{el}$ is known . In fact the actual state of the yield surface is deduced from the initial state by a translation equal to $\underline{A}\ \underline{\Sigma}^{el}$.

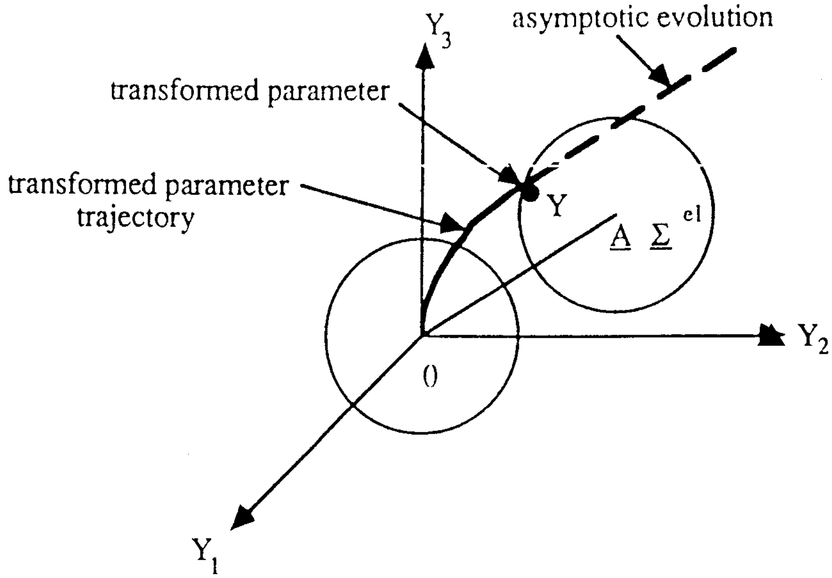


figure 2. Yield surface and transformed parameter evolution.

The limiting state of the material is well-defined by a simple geometrical design. According to the nature of the intersection between the initial and final criteria, C0 and C1, two limiting states can be discriminated. For Von Mises criterion if the distance between the yield surfaces centers is less than twice the Von Mises radius the intersection of the initial and final criteria will not be empty and in this case the material limiting state will be an elastic shakedown, otherwise the limiting state will be a plastic shakedown. In both cases we have defined direct procedures for the choice of $\underline{Y}$ in the limiting state (INGLEBERT et al [1985][1989]) .

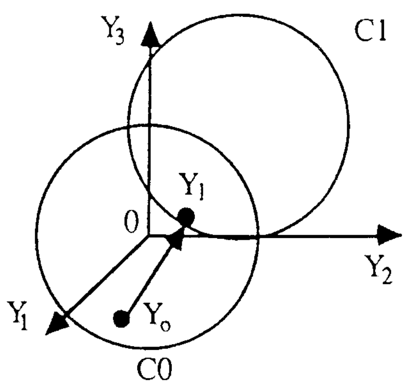


figure 3. Elastic shakedown.

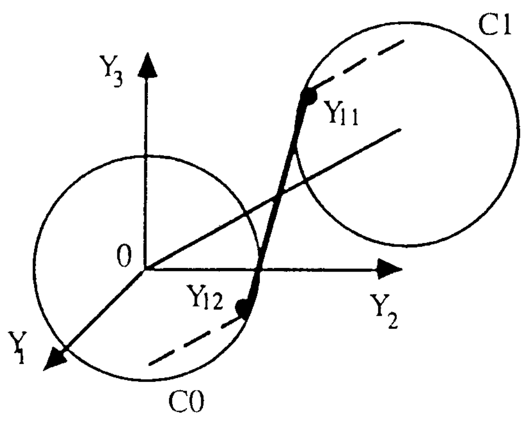


figure 4. Plastic shakedown.
Yl1, Yl2 : estimated bounds of transformed parameter.
------: asymptotic evolution.

III. MODELLING EXAMPLES

III.1. STANDARD MODEL (kinematic hardening)

The elastic-plastic simplified method is used for kinematically hardening materials for which the yield surface will keep its initial shape during the loading. Figure 5.b shows the predicted residual stresses introduced in the material by shot peening. We noted the existence of a constant area in the residual stress profile corresponding to the plastic shakedown zone (strained during loading and unloading), this disagrees with experimental results.

III.2. IMPROVED MODEL (kinematic hardening with isotropic correction)

We have taken into account the isotropic hardening which can be described by a slight increase of the yield surface size at the beginning of the plastic strain. Take ΔR as the increment of the Von Mises radius, we can write:

$$\Delta R_o = k\,(\varepsilon^p)^n \qquad \text{if} \quad 0 \le \varepsilon^p \le \varepsilon^p\text{limit}$$

$$\Delta R_o = k\,(\varepsilon^p\text{limit})^n \qquad \text{if} \quad \varepsilon^p > \varepsilon^p\text{limit}$$

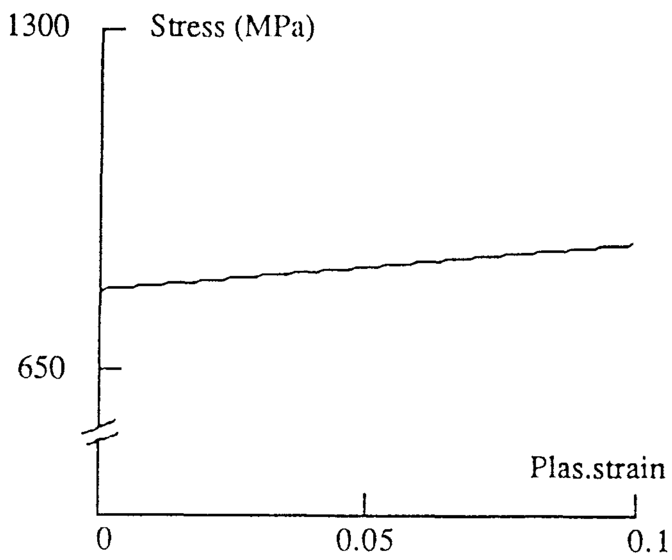
where ε^plimit is the isotropic hardening saturation value, k and n are the classical isotropic hardening coefficents derived from the material cyclic tensile behaviour law .

The effect of isotropic hardening on the theoretical profile is shown in figure 6.b. By introducing this factor we have obtained a better agreement between the experimental and theoretical results in both cases of E 460 steel (equivalent of SAE 1020 steel) and titanium based alloy Ti-6Al-4V . The results are illustrated in figure 7.

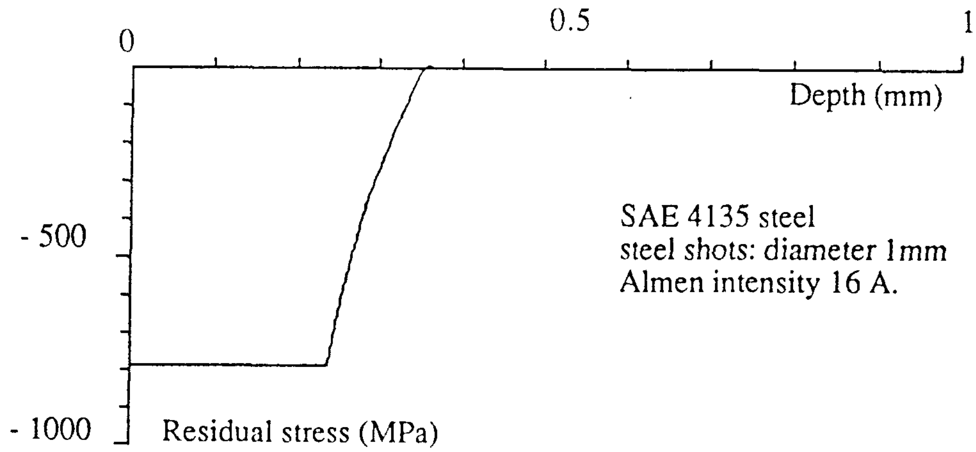
While experimental values for shot peened carbon steels and titanium alloys show a good agreement with the theoretical prediction, results obtained for aluminium alloys are at variance . Figure 8 shows that the predicted affected depth is less than the measured one and the maximal stress is higher than it is in reality. This can be explained by :

- the loading variation due to the difference between the hardness of the shot and the treated material,
- the material behaviour change at high strain velocity
- a bad description of the cyclic behaviour of the material.

We have focused on the third proposition and studied the behaviour of the treated material in more details inorder to make a better approach of it.

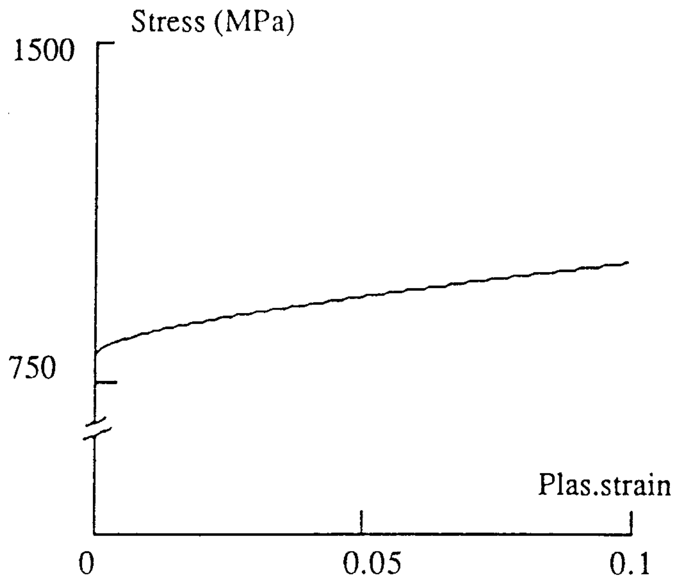


a. Plastic strain-stress curve for a linear kinematic hardening behaviour.

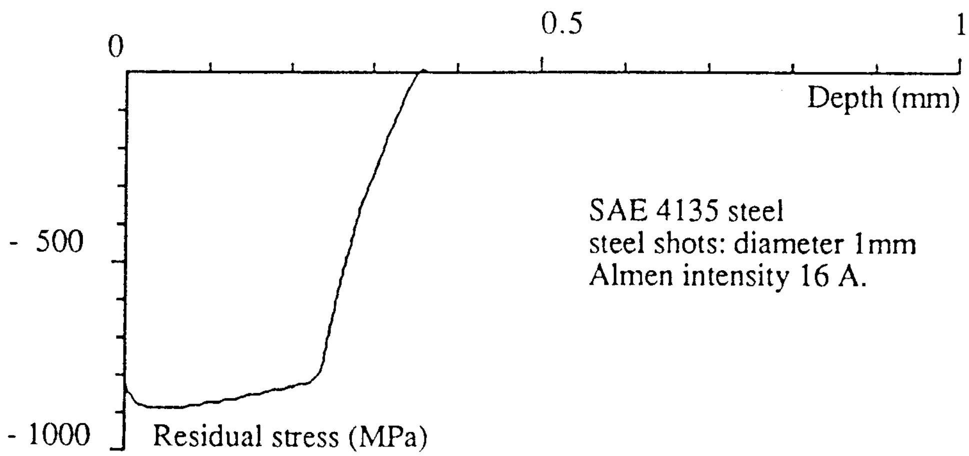


b. Residual stress predicted profile.

Figure 5. Results of standard model for SAE 4135 steel.

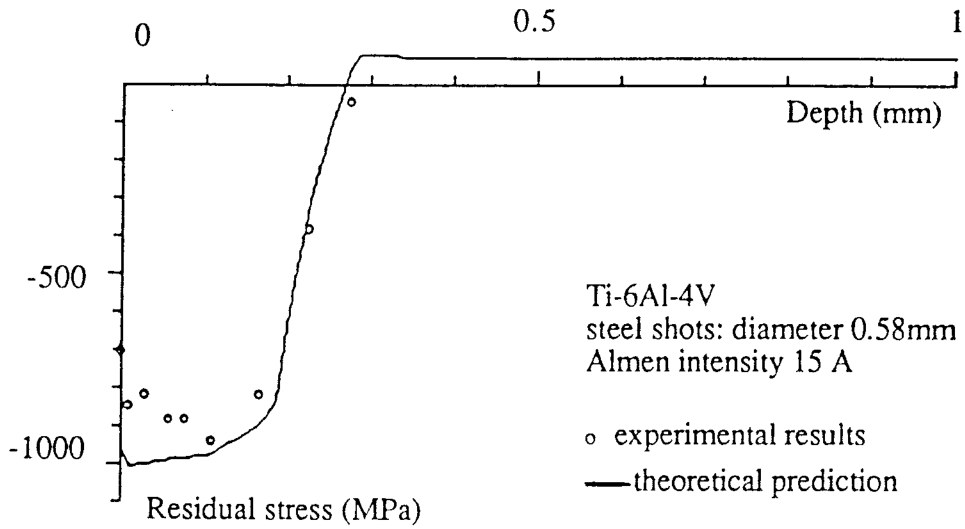


a. Plastic strain-stress curve of a linear kinematic and isotropic hardening behaviour.

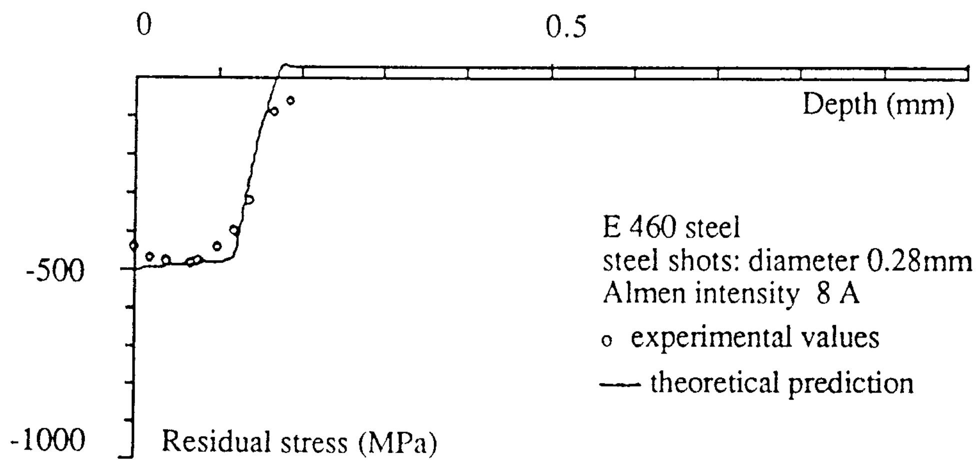


b. Residual stress predicted profile

Figure 6. Results of improved model for SAE 4135 steel.



a. Results for titanium alloy Ti-6Al-4V.



b. Results for SAE 1020 steel

Figure 7. Examples of residual stress profiles predicted with improved model

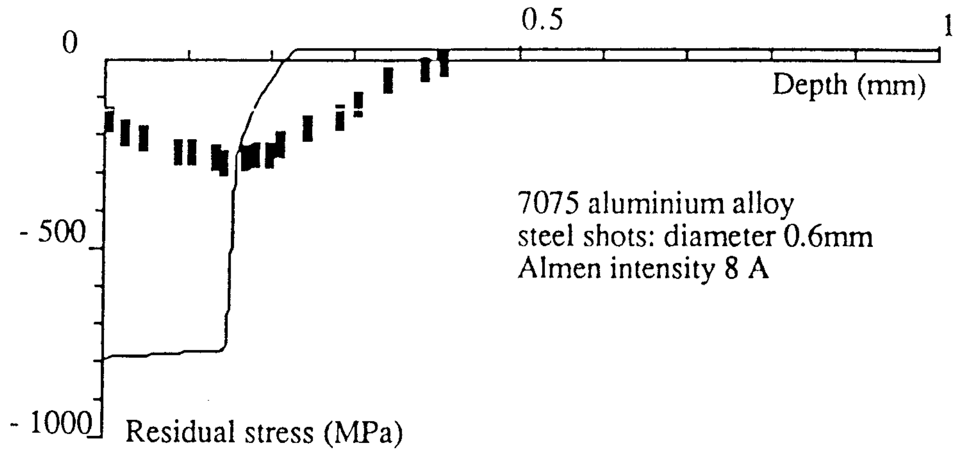


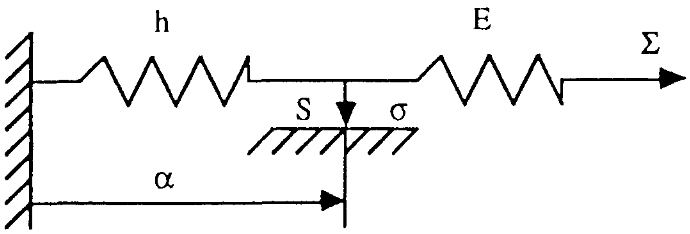
Figure 8. Results for 7075 aluminium alloy
(▮) Experimental and (—) theoretical results.

III.3. NEW APPROACH TO THE PROBLEM (complex hardening)
III.3.1.LINEAR AND MULTILINEAR MODELS

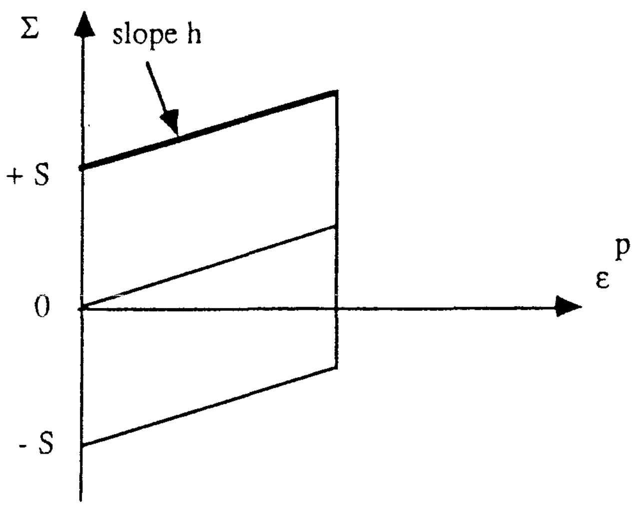
When we compare the theoretical prediction and the experimental measurements of the shot peening residual stresses, a good agreement is obtained for carbon steels and titanium alloys. The difference is however very big for austenitic steels and nickel or aluminium alloys. We think the treated material behaviour law is not very well approached.

The main difference between (carbon steels, titanium alloys) and the second family of materials is related to an important isotropic hardening of the FCC structures. This kind of behaviours cannot be approached by a simple increment of the radius of the criteria surface . We have then focused on a more realistic modelling in the simplified straightforward method framework.

For the kinematic hardening model, the plastic behaviour depends on the evolution of one elementary mechanism of plasticity and can be represented by the rheological model of figure 9. The local stress σ applied to the one-dimensional block must be in the interval (-S,+S), where S is the yielding threshold value.



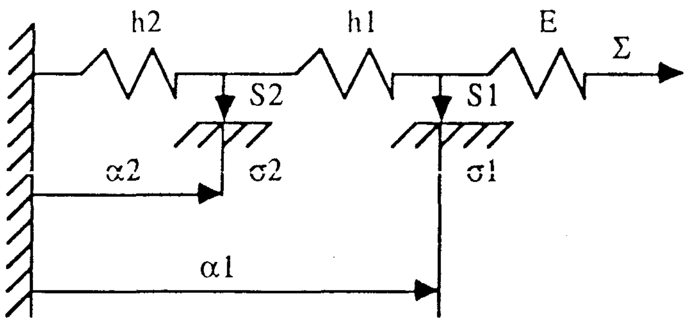
a. linear rheological block.



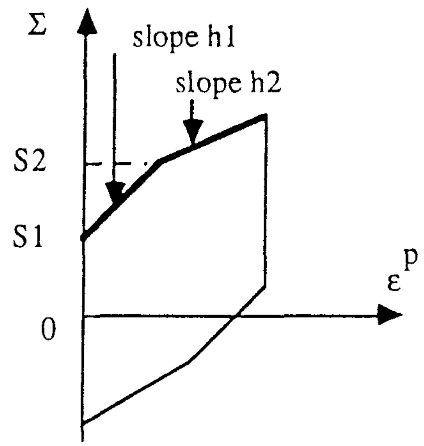
b. Associated loading and unloading curve.

Figure 9. Description of the linear kinematic hardening by a linear block.

We can describe more complicated behaviours by increasing the number of elementary mechanisms, thus we obtain a multilinear description. Figure 10 shows the schematic description of a system composed of two blocks connected in series.



a. Schematic description



b. Associated loading and unloading curve.

Figure 10. Description of the linear kinematic hardening model by two blocks connected in series.

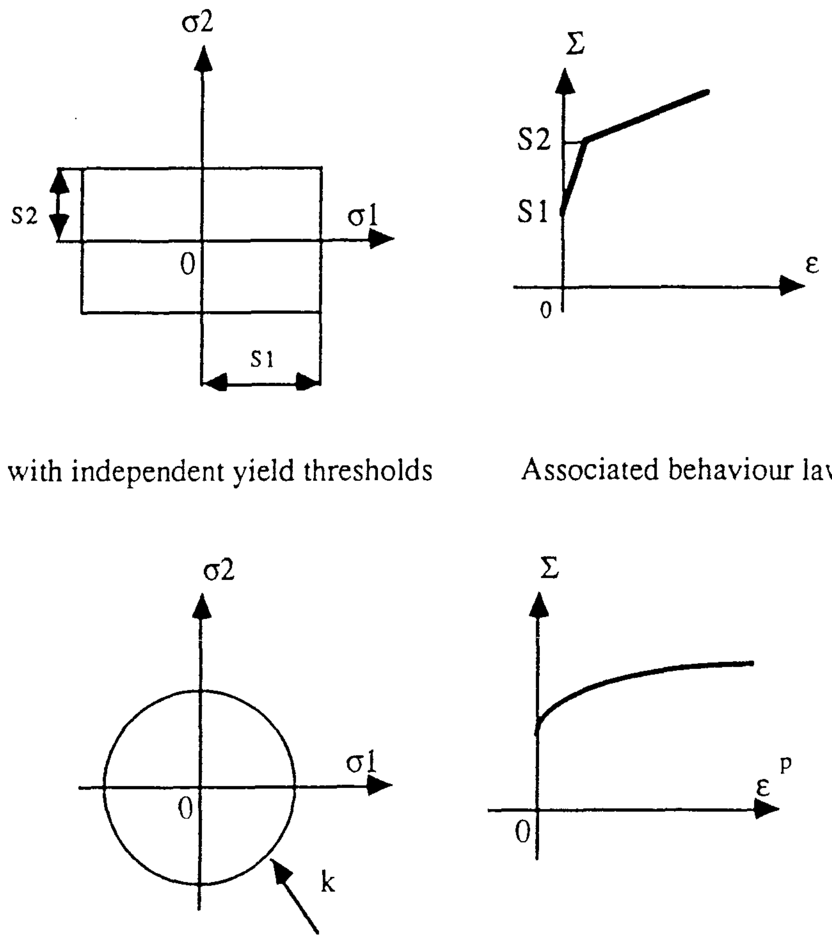
Each of the local stresses σ_1 and σ_2 must be inside its yield threshold interval.

In the (σ_1, σ_2) plane we define a rectangular domain as shown in figure 11.a. The bi-linear aspect of the behaviour law is explained by the successive activation of the internal microscopic mechanisms S1 and S2.

This type of modelling becomes satisfactory when the number of blocks is increased, however if the number of mechanisms is equal to n, the number of the unknowns to find out will be 2n and the problem will become rapidly insolvable. This discrete approach cannot be applied to general cases. So we tried to find out a model giving a continuous description of the threshold evolution with a limited number of parameters.

III.3.2. COUPLED MECHANISMS MODEL

A continuous description of the behaviour law can be obtained by using two blocks having coupled yield threshold values S1 and S2 as represented in the local stress plane (σ_1, σ_2) on Figure 11.b. The coupling allows to reduce the system to only one two dimensionnal block.



a. Blocks with independent yield thresholds — Associated behaviour law

b. Blocks with coupled yield thresholds $S_1^2 + S_2^2 = k^2$ — Associated behaviour law

Figure 11. Description of two blocks system.

Due to global loading Σ^{el} the local stresses σ_1 and σ_2 are transmitted to the blocks by means of their elastic environment (elastic localisation tensor $\underline{A}$). When the yield stress is reached this connected blocks system describes a continuous evolution of the plastic strain-stress curve. If we use this kind of rheological model we must identify four parameters from the real behaviour law [KHABOU 1989]. To take into account the metallurgical evolution due to repetitive loading, we used the classical cyclic behaviour law obtained by reversed tensile loading.

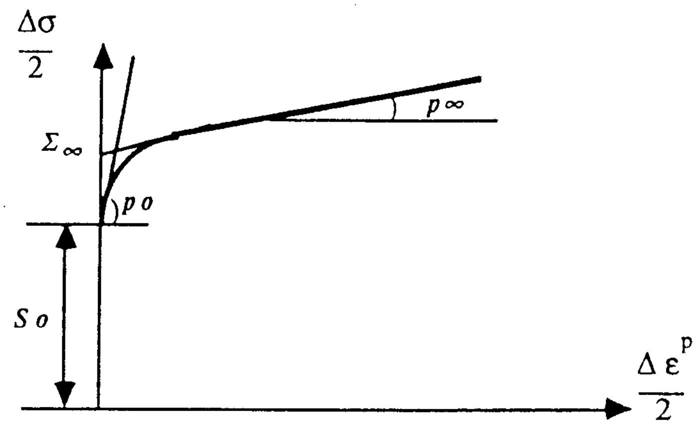


Figure 12. Identification of the parameters

The needed parameters to find out from the real behaviour law are:

- the radius of the initial yield surface S_0,
- the initial slope at the onset of plastic flow p_0,
- the asymptote slope p_∞,
- Σ_∞ the intersection of the asymptote with the stress axis.

Those identified parameters allow at the same time to derive the behaviour law obtained by a coupled yields mechanism (see figure 13.) and to predict the residual stress profile introduced by shot peening (see figure 14.).

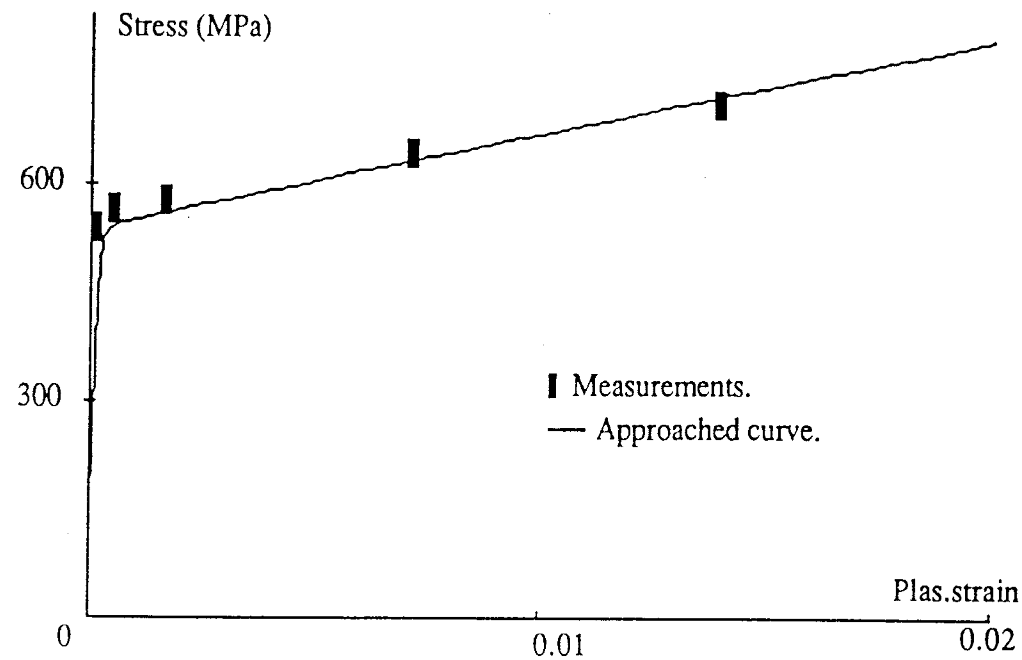


Figure 13. 7075 aluminium alloy behaviour law modelled by a coupled yields block.

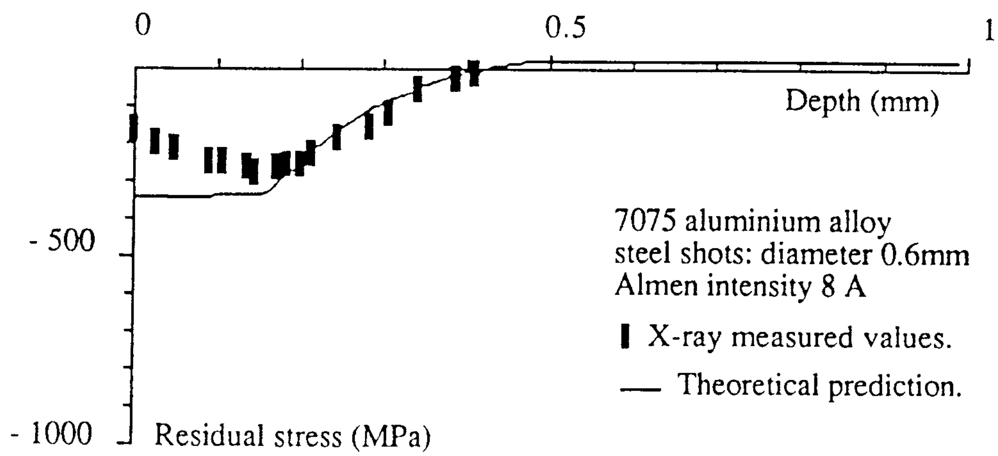


Figure 14. Residual stresses profile introduced in 7075 aluminium alloy.

The profile shown in figure 14. has been obtained for an aluminium based alloy (7075) shot peened by S 230 shots and an Almen intensity of 8 A. Compared to the results of figure 8. the new prediction seems to be satisfactory and needs to be applied for more shot peened materials.

As we have not a significant number (8 would be a minimum) of experimental loops to derive a precise behaviour law we have just done a rough fit of the studied material behaviour law to show the potential performance of the model.

IV. CONCLUSION

The prediction of residual stresses introduced by shot peening is all the better as the knowledge of the exact yield limit of processed material, and its evolution versus the number of cycles or the strain path are more accurate.

For the materials which have a simple flow structure (BCC) the yield stress is definite, a quasi-linear evolution can be observed and elementary modelling could then be sufficient. We note in this case that the shot peening coverage rate has not got a strong influence on the results of the reatment.

The materials with many slip systems (FCC) do not permit the determination of a precise yield stress, moreover they show a complex hardening evolution, in this case we must use a compound rheological assembly. We obtained good results by using an assembly composed of two coupled yields block.

REFERENCES

DAVIES, R.M., 1949, The determination of static and dynamic yield using a steel ball, Proc. Royal. Society A 197, 416.

GUECHICHI, H., CASTEX, L., FRELAT, J. and INGLEBERT, G.,1986, Predicting residual stresses due to shot peening. Impact surface treatment, 11. Elsevier applied science publishers LTD.

INGLEBERT, G., FRELAT, J. and PROIX, J.M., 1985, Structures under cyclic loading, Arch. Mech, 37,4-5, 365, Warszawa.

INGLEBERT, G., FRELAT, J., 1989, Quick analysis of inelastic structures using a simplified method,to be published in Numerical Engineering Design, NED116 (1989) NED 0013X.

JOHNSON, W., 1972, Impact strength of material, Edward Arnolds, London .

KHABOU, M.T., Modélisation du comportement et des contraintes résiduelles introduites dans un matériau soumis à un grenaillage, Thèse de Doctorat, ENSAM Paris, 89 ENAM 0001.

MANDEL,J., 1978, Propriétès mécaniques des matériaux: rhéologie-plasticité, Eyrolles.

SARDAIN, C., 1981, Contribution à l' étude du contact entre deux solides photoélastiques, Doc. Thesis, Université de Poitiers .

ZARKA, J., CASIER, J., 1979, Elastic-plastic response of a structure to cyclic loading, Mechanics today Vol 6 Ed NEMAT-NASSER, Pergamon press.

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