Mechanical approach to the residual stress field induced by shot peening

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Abstract

A simplified analytical model for calculating the compressive residual stress field due to shot peening is given. For eight shot-peening conditions, experimental verifications were carried out on 40Cr steel with four different heat treatment conditions. The results show good agreement with predictions of the model.

1. Introduction

It is well known that shot peening is an effective method to increase the fatigue strength of machine parts. The main mechanism of this effect is regarded as introducing a compressive residual stress field (CRSF) into the surface layer. Many researchers have tried to develop an analytical model for calculating the CRSF according to given shot-peening parameters and target materials. Several models based on elastic-perfectly plastic body have been proposed [1-3]. However, there has been relatively little analytical work on strain-hardening materials. Using the finite element method, Meguid and Klair [4] calculated the CRSF of an elastic linear strain-hardening target peened with incomplete coverage. It may be inferred that when a multilinear hardening material is considered, the calculation must be rather complex.

In this paper a simplified analytical model of the CRSF is developed on the basis of Hertzian elastic contact theory, Iliushin's elastic-plastic theory and the physical concepts of reversed yielding and hardening. A computer block diagram is also given. Finally, experimental verifications are carried out.

2. Theoretical analysis

Shot peening is a process of throwing many small, hard balls with a certain velocity at a target surface. When the velocity of the shot is much less than the speed of sound, dynamic effects can be neglected [1]. Therefore the process may be regarded as a quasi-static one and simulated by statically pressing a hard ball into an elastic-plastic semi-infinite solid.

2.1. Stress-strain analysis of elastic contact

It is assumed that the ball is elastic and has the same $E$ and $v$ as the target. In the coordinate system given in Fig. 1, the stress field in the $Z$ direction is given by hertzian contact theory.

![Coordinate system for stress analysis.](image)
as $|\varepsilon|$  
\[ \sigma_s^e = \sigma_t^e = -q_o[-\frac{1}{2}A + (1 + \nu)B] \]
\[ \sigma_z^e = -q_o A \]
where
\[ A = \left[ 1 + \left( \frac{\sigma}{a} \right)^2 \right] \]
\[ B = 1 - \frac{z}{a} \tan^{-1} \left( \frac{a}{z} \right) \]
\[ q_o = \left( \frac{6FE}{4\pi R^2 (1 - \nu^2)} \right)^{1/3} \]
\[ a = \left( \frac{3}{4} \frac{RF}{E} \right) \]
\[ 2(1 - \nu^2) \]
in which $F$ is the loading, $q_o$ is the maximum pressure at the contact centre, and $a$ is the radius of the circular contact area; $\sigma_s^e$, $\sigma_t^e$ and $\sigma_z^e$ are the principal stresses. Hence
\[ \sigma_s^e = \frac{1}{2} \left[ (\sigma_s^e - \sigma_t^e)^2 + (\sigma_s^e - \sigma_z^e)^2 + (\sigma_z^e - \sigma_t^e)^2 \right]^{1/2} \]
\[ = q_o [\frac{1}{2}A - (1 + \nu)B] \]
\[ \sigma_t^e = \frac{1}{2} (\sigma_s^e + \sigma_t^e + \sigma_z^e) \]
\[ = -q_o (1 + \nu)B \]
\[ \sigma_z^e = \sigma_z^e - \sigma_m^e \]
\[ = \frac{1}{4} q_o [A - (1 + \nu)B] \]
\[ \sigma_t^e = \sigma_t^e - \sigma_m^e \]
\[ = \frac{1}{4} q_o [A - (1 + \nu)B] \]
\[ \sigma_z^e = \frac{\sigma_z^e}{E} \]
\[ e_s^e = e_t^e = \frac{1}{E} \left[ (\sigma_s^e - \nu(\sigma_s^e + \sigma_t^e)) \right] \]
\[ = -\frac{1 + \nu}{E} q_o [A - (1 + \nu)B] \]
\[ e_z^e = \frac{1}{E} (\sigma_z^e - 2v\sigma_t^e) \]
\[ = -\frac{1 + \nu}{E} q_o [A - 2vB] \]

![Fig. 2. Schematic diagram for calculating transresidual stress.](image)

Similarly, the strain deviations are
\[ e_s^e = e_t^e = \frac{1}{2} (1 + \nu)\varepsilon_s^e \]
\[ e_z^e = -\frac{1}{2} (1 + \nu)\varepsilon_z^e \]
\[ = -2\varepsilon_z^e \]

2.2. Elastic-plastic analysis of the loading process

The stress-strain relationship of a target material can be simplified to a multilinear one (Fig. 2) in which $\sigma_o$ is the true stress corresponding to the maximum stress in an engineering stress-strain curve, $\sigma_o$. On the basis of the single-curve hypothesis, this curve can be regarded as a $\sigma$ vs $\varepsilon$ curve.

When $\sigma_o > \sigma$, the target material enters the elastic-plastic deformation stage; the stress-strain analysis becomes very complex and is not convenient to apply in practice. Therefore in this paper a simplified method was used, i.e., we calculated the elastic-plastic strain $\varepsilon^p$ approximately from the elastic strain $\varepsilon^e$ by adopting a modifying coefficient $\alpha$:

\[ \varepsilon^p = \begin{cases} 
\varepsilon^e & \text{for } \varepsilon^e < \varepsilon_s, \\
\varepsilon_s + \alpha(\varepsilon^e - \varepsilon_s) & \text{for } \varepsilon^e > \varepsilon_s 
\end{cases} \]
Here $\alpha$ is the ratio of plastic to elastic deformation and is defined as

$$\alpha = \frac{D_p}{D_L}$$

in which $D_L = 2a$ (eqn. (3)) and $6$

$$D_p = \beta E^u$$

where

$$\beta = (16.00 + 15.00 R) + 1.66(1 + 2R)$$

$$n = 0.482(1 - 0.2R)$$

$D_L$ is the diameter of a dent produced by pressing the same ball on an elastic-plastic body under the same load as in eqn. (3) and then unloading. In fact, there is a hypothesis in eqn. (8) that the ratio of $e_p^t$ to $e_p^c$ on the $Z$ axis inside the target is equal to $\alpha$, i.e. the ratio of deformation at the surface.

According to the $\sigma_i$ vs. $\epsilon_i$ curve, $\sigma_p^t$ is obtained as

$$\sigma_p^t = \begin{cases} 
\sigma_i^c & \text{for } \epsilon_p^c < \epsilon_i^c \\
\sigma_i^c + (\epsilon_p - \epsilon_i^c)k_1 & \text{for } \epsilon_i^c < \epsilon_p < \epsilon_b \\
S_b + (\epsilon_p^b - \epsilon_b)k_2 & \text{for } \epsilon_p^b \geq \epsilon_b
\end{cases}$$

(11)

The definitions of $k_1$, $k_2$, $\epsilon_i^c$, $\epsilon_p$, $\sigma_i$ and $S_b$ are shown in Fig. 2. Similarly, a more complex calculation can be carried out for a more nonlinear hardening material.

In the elastic stage there is a relationship between $\epsilon_i^c$, $\epsilon_i^t$, $\epsilon_p^c$ and $\epsilon_p^t$ as in eqn. (7). Since this relationship results from the axisymmetry of loading and geometric conditions, it must be kept in the elastic-plastic stage as

$$\epsilon_p^c = \epsilon_i^c = \frac{1}{2}(1 + v)e_i^t$$

$$\epsilon_p^t = -2\epsilon_i^t$$

According to Iliushin's elastic-plastic theory $7$,

$$s_p^t = \frac{1}{1 + v} \frac{\sigma_p^t}{\epsilon_p^t}$$

Then

$$s_p^t = \frac{1}{1 + v} \frac{\sigma_p^t}{\epsilon_p^t} = \frac{1}{2}\sigma_i$$

$$s_p^t = \frac{1}{2}\sigma_i$$

$$s_p^t = -\frac{1}{2}\sigma_i$$

(13)

2.3. Calculation of residual stress after unloading

It is assumed that (1) the amount of deformation is small, (2) unloading is an elastic process until reversed yielding starts and (3) hydrostatic stress does not introduce plastic deformation. Under these conditions the basic formula for calculating the residual stress is

$$\sigma_{i}^t = s_{i}^p - s_{i}^c$$

(14)

Assuming the target material is an isotropic one, then

$$\sigma_{i}^t = \begin{cases} 
0 & \text{for } \sigma_i^c < \sigma_i \\
(\sigma_i^p - \sigma_i^c) & \text{for } \sigma_i \leq \sigma_i^c \leq 2\sigma_i^p
\end{cases}$$

(15)

Thus

$$\sigma_i^t = \sigma_i = \frac{1}{2}(\sigma_i^p - \sigma_i^c)$$

(16)

When $\sigma_i^c > 2\sigma_i^p$, reversed yielding and hardening must take place. In this case the calculation of $\sigma_i^t$ is schematically shown in Fig. 2. First a stress of $2\sigma_i^p$ is elastically unloaded, then reversed yielding takes place, but there are still some stresses which have not been unloaded, namely

$$\Delta \sigma_i^t = \sigma_i^t - 2\sigma_i^p$$

(17)

The elastic and elastic-plastic strains corresponding to $\Delta \sigma_i^t$ are

$$\Delta \epsilon_i = \frac{\Delta \sigma_i^t}{E}$$

(18)

Then the corresponding stress $\Delta \sigma_i^p$ can be obtained using the $\sigma_i$ vs. $\epsilon_i$ curve as in Fig. 2; thus

$$\sigma_i^t = \sigma_i^t = \frac{1}{2}(\sigma_i^p - 2\sigma_i^p - \Delta \sigma_i^p)$$

(19)

$\sigma_i^t$, $\sigma_i^t$, and $\sigma_i^t$ are only the residual stresses after loading and unloading a single ball and are called transresidual stresses. After shot peening with 100% coverage it is assumed that the plastic deformation is steady and continuous; the solid will retain a plane surface after full-coverage peening, so that $\epsilon_i^t$ and $\epsilon_i^t$ must be zero and the
non-zero stress and strain components will be independent of \(x\) and \(y\). The only possible system of residual stresses and strains, therefore, will be

\[
\sigma_{i}^r = \sigma_{i} = f(z), \quad \sigma_{i}^e = 0
\]

\[
\varepsilon_{i}^r = \varepsilon_{i} = f(z), \quad \varepsilon_{i}^e = 0
\]

However, the transresidual stresses and strains do not satisfy these conditions of equilibrium and must be partially relaxed. The relaxation values of \(\sigma_{i}^r\) and \(\sigma_{i}^e\), i.e. \(\sigma_{i}^{r*}\) and \(\sigma_{i}^{e*}\), can be calculated in accordance with Hooke's law as

\[
\sigma_{i}^{r*} = \sigma_{i}^{e*} = \frac{v}{1 - v} \sigma_{i}^e
\]

Thus, after deducting the relaxation values, the final residual stresses \(\sigma_{i}^{r*}\) and \(\sigma_{i}^{e*}\) are [8]

\[
\sigma_{i}^{r*} = \sigma_{i}^{r*} = \sigma_{i}^r - \frac{v}{1 - v} \sigma_{i}^e
\]

\[
= \frac{1 + v}{1 - v} \sigma_{i}^r
\]

2.4. Computer programme

With the above theory, it is very convenient to calculate the residual stress due to shot peening on a personal computer. For an isotropic multilinear hardening material the flow diagram is given in Fig. 3. The programme can be used for a double-linear hardening or an elastic (rigid)-perfectly plastic material. With minor modifications it is easy to compute the CRSF of a kinematic hardening target. Moreover, if the Bauschinger effect was measured out by the tension and compression test, its data could be input to the programme and the calculation precision thus improved.

3. Experimental verification and CRSF construction analysis

3.1. Experimental verification

The chemical composition of the 40Cr steel investigated in this paper was 0.41C-0.72Mn-0.19Si-0.030P-0.009S-1.0Cr-0.08Ni (weight per cent). Tension specimens with a gauge size of 10 mm diameter by 100 mm length were austenitized in a salt bath at 840°C for 10 minutes, quenched in oil and then tempered for 2 h at 200, 400, 550 and 650°C respectively. Their tension true stress–true strain curves were multilinearized and are shown schematically in Fig. 3. Their mechanical properties are given in Table 1. After being heat treated under the same conditions, specimens for shot peening (size 20 × 20 × 10 mm³) were annealed at 200°C for 48 h and then mechanically and electrolytically polished to remove a layer of about 100 μm from the surface to ensure that any machining or heat treatment effect which could mask the results was absent. The shot peening with 100% coverage was
TABLE 1
Parameters for calculating the CRSF

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$E$ (10$^3$ MPa)</th>
<th>$v$</th>
<th>$\sigma_0$ (MPa)</th>
<th>$\epsilon_c$ ($\times 10^{-4}$)</th>
<th>$S_c$ (MPa)</th>
<th>$\epsilon_c$ ($\times 10^{-4}$)</th>
<th>$d$ (mm)</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>2.0</td>
<td>0.3</td>
<td>1420</td>
<td>7.1</td>
<td>1950</td>
<td>35.0</td>
<td>1.10</td>
<td>2</td>
</tr>
<tr>
<td>A2</td>
<td>2.0</td>
<td>0.3</td>
<td>1420</td>
<td>7.1</td>
<td>1950</td>
<td>35.0</td>
<td>0.55</td>
<td>1</td>
</tr>
<tr>
<td>B1</td>
<td>2.0</td>
<td>0.3</td>
<td>1270</td>
<td>6.4</td>
<td>1540</td>
<td>45.0</td>
<td>1.10</td>
<td>3</td>
</tr>
<tr>
<td>B2</td>
<td>2.0</td>
<td>0.3</td>
<td>1270</td>
<td>6.4</td>
<td>1540</td>
<td>45.0</td>
<td>0.55</td>
<td>1</td>
</tr>
<tr>
<td>C1</td>
<td>2.0</td>
<td>0.3</td>
<td>980</td>
<td>4.9</td>
<td>1210</td>
<td>60.0</td>
<td>1.10</td>
<td>3</td>
</tr>
<tr>
<td>C2</td>
<td>2.0</td>
<td>0.3</td>
<td>980</td>
<td>4.9</td>
<td>1210</td>
<td>60.0</td>
<td>0.55</td>
<td>1</td>
</tr>
<tr>
<td>D1</td>
<td>2.0</td>
<td>0.3</td>
<td>700</td>
<td>3.5</td>
<td>885</td>
<td>140.0</td>
<td>1.10</td>
<td>2</td>
</tr>
<tr>
<td>D2</td>
<td>2.0</td>
<td>0.3</td>
<td>700</td>
<td>3.5</td>
<td>885</td>
<td>140.0</td>
<td>0.55</td>
<td>1</td>
</tr>
</tbody>
</table>

diameter $D_p$ was obtained. Finally, in accordance with eqn. (10), $F$ was calculated and its values are listed in Table 1.

The macroscopic residual stresses produced by peening were measured on a 2903-type X-ray diffractometer with the 0°-45° method, selecting Cr Kα (211) interference lines. The peak of the diffraction intensity curve was determined by the half-width method. In order to obtain the residual stress distribution curve along the depth direction, the surface layer was removed step by step using electrolytical polishing. Since the errors produced by layer removal are small enough to neglect, the measurement results were not corrected. The measured results are shown in Fig. 5.

Calculation of the CRSF was performed using the programme JKG-88, which was drawn up on the basis of the theoretical analysis model of this paper. The calculated results are compared with the experimental results in Fig. 5; they show good agreement.

3.2. CRSF construction analysis

According to Fig. 6, in which the $\sigma^f$ vs. $Z$, $\epsilon$ vs. $Z$ and $\sigma^R$ vs. $Z$ curves of specimen B1 (Table 1) are drawn, the CRSF can be divided into four regions.

3.2.1. Region A

In this region the target material deforms elastically during unloading. The controlling conditions for region A are $\sigma^f > \sigma_c$ and $\sigma^f < 2\epsilon_c$. Since in this region the gradient of $\sigma^R$ is large, the low surface residual stress $\sigma^R$ is produced. However, when the load is large enough or the hardness of the target is lower, region A may be absent; then $\sigma^R$ is higher. It is well known that fatigue tests cracks may nucleate either on
surface or in the tensile residual stress area, depending on the relationship between the local resistance of the metal to fatigue and the sum of residual and externally applied stresses. According to the CRSF calculated here, the position and value of the peak tensile residual stress can be inferred [9], the position where cracking will take place can be determined.

3.2.2. Region B

In this region reversed yielding and hardening occur. The controlling condition for region B is \(2\sigma_P^r < \sigma_f^r\). In this region the stress gradient is lower but the residual stress values are high. It is this region that retards the nucleation and propagation of fatigue cracks. By this action the position of crack nucleation is generally moved from the surface to the subsurface layer and the fatigue strength is increased.

3.2.3. Region C

Reversal hardening of the metal will reach saturation degree in this region. This region is included in region B; the additional controlling condition can be written as \(\sigma_P^p + \Delta \sigma_P^p \geq S_h\) or, more exactly, \(\sigma_P^p + \Delta \sigma_P^p \geq S_h\). In practice, shot peening does not induce a strain much more than \(\epsilon_h\) (the strain at \(S_h\)); furthermore, the \(\sigma\) vs. \(\epsilon\) curve becomes more gentle after point \(S_h\), so it is reasonable to take \(S_h\) as the maximum value of the \(\sigma\) vs. \(\epsilon\) curve. Then the equation

\[
\sigma_{\text{max}}^p = \frac{1 + \nu}{2(1 - \nu)} S_h \quad \text{(for } k_3 = 0) \tag{21}\]

can be used to predict the maximum value of the residual stress produced by peening. According to this formula, the values of \(\sigma_{\text{max}}^p\) for the four kinds of specimens investigated are equal to 1210, 955, 750 and 549 MPa (for \(\nu = 0.3\)) respectively, which coincide with the experimental results quite well. In order to obtain an optimum fatigue resistance, the peening parameters should be selected so that the peak of the CRSF reaches \(\sigma_{\text{max}}^p\). The minimum peening condition which just satisfies this situation is defined as the critical peening condition. When the peening condition exceeds the critical one, a wide region C (also region B) will be produced. In this case, although the wide compressive residual stress region can retard fatigue crack propagation, the seriously damaged surface may cause cracks to nucleate earlier, thus shortening the fatigue life. Now it can be concluded that the optimum CRSF must have a high surface residual stress and a deep and narrow region C. This point of view has been supported by some of our experimental results [9].

3.2.4. Region D

Here elastic unloading takes place just as in region A. However, in contrast to region A, region D always exists and its shape depends only on the mechanical properties of the target (only if the critical peening condition has been satisfied). In this region the stress gradient is higher than in region B and generally lower than in region A. The internal boundary of the region depends on \(\sigma_i\) of the target, so the lower \(\sigma_i\) of the target is, the wider region D is. Compared with regions B and C, this region does not make an important contribution to increasing the fatigue strength of the target, but the region is very useful for calculating the tensile residual stress field which is used to calculate the local fatigue strength.

Finally, it should be pointed out that the model in paper does not give the tensile residual stress field and is only suitable for shot peening with 100% coverage. These two points will be considered in future work.
References


Appendix A: Nomenclature

- $d$: average diameter of shot
- $\varepsilon$: strain deviation
- $\varepsilon_m$: mean strain
- $\varepsilon_i$: effective strain
- $\sigma$: true stress
- $\sigma_m$: mean stress
- $\sigma_i$: effective stress
- $\sigma'$: transresidual stress
- $\sigma_R$: residual stress after shot peening
- $\sigma_{\text{max}}^R$: maximum value of residual stress
- $\sigma^R$: surface residual stress

Greek letters

- $\varepsilon$: true strain
- $\sigma$: Young's modulus
- $F$: loading
- $R$: radius of ball
- $s$: stress deviation

Superscripts

- $e$: calculated by elastic theory
- $p$: calculated by elastic-plastic theory