SHOT-PEENING MECHANICS, 
A THEORETICAL STUDY

Dr S Kyriacou, 
Engineering Mechanics Group, School of Mechanical Engineering, 
Cranfield University, Cranfield, Bedford, UK

ABSTRACT

The mechanics of shot-peening is considered from the perspective of the Hertzian theory, and the requirement for modelling with theoretical/numerical methods are discussed. The modelling scheme developed enables the investigation of single and multiple indentations and provides predictions of the residual stress field and depth of work hardening. The results have shown that the strain hardening behaviour of the target material influences the induced residual stressess, and incomplete coverage induces beneficial compressive residual stresses or in some cases tensile, that explain the detrimental effects that have been observed in cases of poor coverage.

INTRODUCTION

Surveys into the causes of in service failures carried out over the last twenty years attribute approximately 80% of catastrophic fractures to fatigue, which consequently has been identified as the largest single cause of failure in structures and machine systems. The problems associated with the behaviour of metals under repeated stresses are becoming more acute with increases in the operational demands on machinery. Design strategies that postulate high mechanical efficiency and adequate static and dynamic strengths, for minimum structural weight, are becoming mandatory in view of the rising costs of materials and energy. Such strategies dictate the optimum use of materials that in turn demand sound understanding of the material behaviour under operational loading conditions. In many cases use is made of post-machining treatments, and their effects must be quantified and accounted for at the design stage.

Treatments of thermal or thermochemical nature have limited applicability as they appear to discriminate on the material type (ferrous, non-ferrous). Mechanical treatments do not discriminate on the type of material, but their drawbacks are in terms of component geometry and shape. One mechanical surface treatment has been found to be, not only nondiscriminatory but also highly versatile and adaptable. This is the impact surface treatment commonly known as shot peening, which is characterized by versatility, high productivity rates, a notably modest equipment cost, and low energy requirements.

Employing shot peening to improve the structural integrity of metallic components, necessitates thorough understanding of the mechanics of the process. The mechanism of protection against fatigue failure contributed by impact treatments is widely attributed to cold work and accumulation of compressive residual surface and sub-surface stresses, which must be sufficiently large, compressive and their distribution uniform and continuous in order to suppress the initiation of surface cracks.
This study deals with the mechanics of the shot peening process, adopting a theoretical/numerical approach, that provides the means for predicting residual stresses whilst accounting for a number of variables influencing the process.

IMPACT SURFACE TREATMENT

Shot-peening is a cold working surface treatment widely used in the aerospace, automotive, gas turbine, as well as pump and power industries primarily for improvement of the fatigue structural integrity of metallic components. This is accomplished by bombarding the surface of metallic materials with small spherical shots made of cast high carbon steel, iron, conditioned cut-wire, glass or ceramic projected with high impinging velocities. Each shot acts as a peen-hammer and impact with the exposed surface produces localised stretching of the surface layer and cold working to a depth of 120-500μm. Upon completion of the process, a compressive residual stress field is left on the workpiece that has been found to be highly effective in preventing premature failure of components subject to cyclic loads [1, 2].

The beneficial effects of shot-peening depend upon the work hardening of the surface layer and formation of compressive residual stresses, caused by the bombardment of the workpiece with shot. It has been found that great benefits can be obtained if the intensity of residual stresses and the depth of penetration of work hardening are maximised. Evidently, the effectiveness of the shot-peening process is dependent upon the energy transfer that occurs during the impact of shots with the target surface and the uniformity of the induced compressive residual stresses. In practice the process efficiency is evaluated by means of intensity, saturation and coverage [3,4].

Intensity, correlates the amount of energy being transferred during the impact of a typical shot with the workpiece and it is related to the kinetic energy of the blast stream.

Saturation, deals with the number of particles that impinge upon the target, which can be manipulated by means of altering the exposure time.

Coverage, is a means of quantifying the uniformity of residual stresses by visually examining a small area of the treated surface.

Clearly, a large number of variables control the efficiency of shot peening. These can be classified in three groups: shot, target and flow parameters, as follows:

i. Shot Parameters:
   Size, shape, integrity, density, hardness, yield strength, modulus of elasticity...

ii. Target Parameters:
   Hardness, yield strength, modulus of elasticity, work hardening characteristics, material chemical composition, prestress condition...

iii. Flow Parameters:
   Mass flow rate, velocity, angle of impingement, stand-off distance...
CONTACT MECHANICS FUNDAMENTALS

Shot peening mechanics is a complex problem due to the large number of variables involved and the nonlinear nature of impact-contact phenomena and associated elasto-plasticity. However, some understanding of the process can be gained by considering the ideas of contact mechanics and in particular the available solution associated with the axisymmetric indentation of an elastic surface by a spherical indenter.

Explicit theoretical solutions for the contact of elastic bodies were developed by Hertz and presented in his pioneering work considered as the foundation of contact mechanics [5]. The Hertzian elastic solution shows that the contact area between two elastic axisymmetric bodies is a circle whose projected radius is given by the expression,

\[ a = \left[ \frac{3QR}{4 \left( \frac{1}{E^*} \right)} \right]^{\frac{1}{3}} \]  

(1)

where \( Q \) is the applied load, \( E^* \) is the equivalent contact modulus and \( R \) the relative radius of curvature respectively defined as,

\[ \frac{1}{E^*} = \left( \frac{1-v_1^2}{E_1} \right) + \left( \frac{1-v_2^2}{E_2} \right) \quad \text{and} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \]  

(2)

The Hertzian pressure distribution within the circular contact area is parabolic with the maximum pressure \( P_{\text{max}} \) present at the centre of the indentation. The contact pressure \( P(r) \) varies along the indentation radius according to,

\[ P(r) = P_{\text{max}} \sqrt{\left( 1 - \left( \frac{r}{a} \right)^2 \right)} \quad \text{and} \quad P_{\text{max}} = \left( \frac{3}{2} \right) \frac{Q}{\pi a^2} = \left( \frac{6 Q E^*}{\pi^3 R^2} \right)^{\frac{1}{3}} \]  

(3)

The surface stresses (z=0), generated by the elastic spherical indenter both inside and outside the circular contact area can be evaluated by the following expressions [6],

\[ \frac{\sigma_r}{P_{\text{max}}} = \left( \frac{1-2v}{3} \right) \left( \frac{a}{r} \right)^2 \left\{ 1 - \left[ 1 - \left( \frac{r}{a} \right)^2 \right] \right\} - \sqrt{1 - \left( \frac{r}{a} \right)^2} \]  

\[ \frac{\sigma_\theta}{P_{\text{max}}} = \left( \frac{2v-1}{3} \right) \left( \frac{a}{r} \right)^2 \left\{ 1 - \left[ 1 - \left( \frac{r}{a} \right)^2 \right] \right\} - 2v \sqrt{1 - \left( \frac{r}{a} \right)^2} \]  

\[ \frac{\sigma_z}{P_{\text{max}}} = \sqrt{1 - \left( \frac{r}{a} \right)^2} \]  

(4)
Furthermore, for the axisymmetric elastic contact case expressions (5) have been developed describing the sub-surface stress distribution [7].

\[
\frac{\sigma_r}{P_{\text{max}}} = \frac{\sigma_z}{P_{\text{max}}} = -(1+\nu) \left\{ 1 - \left( \frac{z}{a} \right) \tan^{-1} \left( \frac{a}{z} \right) \right\} + \frac{1}{2} \left[ 1 + \left( \frac{z}{a} \right)^2 \right]^{-1}
\]

The solution for the contact between a spherical indenter and an elastic surface is incapable to provide the information relevant to shot-peening, i.e. residual stresses (magnitude, distribution), depth of penetration of the plastic zone, depth of work hardening. This is due to the assumption made by Hertz that the contact is elastic. Further, the solution is axisymmetric and as such it cannot deal with multiple or oblique indentations. However, the elastic theory provides fundamental information that could be utilised to enhance understanding of the shot peening mechanics.

SHOT PEENING MODELLING
The aim of modelling is to produce a parametric representation able to adequately delineate the shot peening process, and

- provide quantitative information (residual stress distribution and magnitude...)
- allow changes in variables (velocity, shot size, shape, angle of impingement...)
- accommodate material properties (elastic, elasto-plastic, strain hardening...)

Over the last fifteen years several modelling efforts have been reported in the technical literature. In all cases the developed models aim to predict the residual stresses, utilising suitable theoretical or numerical schemes [8-11]. Although, some of the models have been validated against experimental measurements and show good agreement, they assume the indenter to be perfectly spherical and rigid [8,11].

New computational models have been developed aiming to enhance the modelling capabilities already available, and advance the understanding of the mechanics of the shot peening process. The models are based upon the use of Finite Element Methods and take advantage of the availability of interactive pre-processing software, and highly specialised solvers designed specifically for the solution of nonlinear problems in contact and elasto-plasticity [12-15]. The models developed enable the analysis of single and multiple indentations. In the most simplified form a model can deal with a single ball indentation in quasi-static conditions and do not account for the presence of the indenter, represented by an equivalent pressure distribution applied on the contact area which is predetermined by considering equations (1-5).
The target material can be modelled as an elasto-plastic continuum with varying strain hardening characteristics, linear or kinematic strain hardening. A finite element model is designed to provide a suitable fine and focused mesh in the area of contact. Noting that the single indentation problem is symmetric along the axis of load application, for normal contact, and taking advantage of the exhibited symmetry, a half, quarter or smaller sector of the geometry can be used. An obvious benefit of the modelling method is that it can be modified in order to model multiple indentations. This is achieved by modelling half the contact area i.e. a quarter of the whole system. Typical mesh configurations for the study of single and multiple indentations are depicted in Fig.1.

Figure 1  Typical Finite Element Meshes for Single and Multiple Indentations

Detail information on the finite element based modelling of the shot peening process developed by the author, can be found in [15], which includes an extensive account of methodology, different types of models, and experimental verification.

Elastic Analysis

The elastic sub-surface shear stress distribution obtained from a typical elastic finite element solution is shown in Fig.2, and is compared with the Hertzian theory predictions and photo-elasticity results. Model predictions are evidently in very good agreement with theory and experiment. The results of photo-elasticity exhibit the overall trend, however, they give higher values of non-dimensional shear stress \( \left( \frac{T_{\text{max}}}{P_{\text{max}}} \right) \) and location \( (z/a) \) of the maximum value. This has been attributed to experimental measurement errors, as the isochromatic fringes were very closely packed in the vicinity of contact, making it difficult to identify their order and exact location.
Figure 2 Comparison of Finite Element, Theoretical, and Experimental Elastic Results

Elasto-Plastic Analysis

Investigating the shot-peening process inevitably requires prediction of the residual stresses developed in the workpiece for a given set of parameters. This could be achieved by means of a solution capable of accounting for material elasto-plasticity. An investigation of the effect of workpiece material behaviour on the development of residual stresses, due to a given shot size and velocity, considered a medium carbon steel with, $E=210.0\text{GPa}$, $\nu=0.30$ and $\sigma_{\text{yield}}=600.0\text{MPa}$. Idealised post yield material models were used, perfectly-plastic ($H=0.0$) and linearly strain hardened with $H=0.1$ and $H=0.2$. The term $H$ is defined as the strain hardening modulus, $H=E_t/E$, where $E_t$ is the tangent and $E$ the elastic modulus respectively. This examined the residual stress depth and distribution as a function of workpiece material hardening and indenter separation.

The modelling method used in this analysis of the single and multiple elasto-plastic indentations, is that of 3D Finite Elements without the indenter Fig.1. The loading was represented by an idealised Hertzian pressure distribution, and was applied incrementally so that the first load increment was sufficient to bring the material to the threshold of plasticity, $\tau_{\text{max}}=0.95\sigma_{\text{yield}}$. Subsequent load increments raised the load in steps of no more than 10% of the final load, whilst the material was undergoing plastic deformation. Application of the full load $3.3\sigma_{\text{yield}}$ was followed by single step unloading.

The residual $\sigma_{xx}, \sigma_{yy}$ stress distributions along the $z$-axis in the centre of the single indentation are shown in Fig.3, for the three strain hardening models used, whilst definitions of the directions are defined in Fig.1. Clearly, the residual $\sigma_{xx}, \sigma_{yy}$ stresses are
compressive on the surface and become more intense below the surface. These results compare favourably with experimental observations [2]. From the data depicted it is apparent that the strain hardening modulus affects the residual stress distribution. In fact the higher the value of H the lower the peak compressive residual stress Table 1.

![Graph showing residual stress distribution](image)

**Figure 3** $\sigma_{xx}$ Residual Stress Distribution at the Centre of a Single Indentation

**Table 1** Effect of Strain Hardening on the Residual Stress Distributions due to a Single Indentation

<table>
<thead>
<tr>
<th>H</th>
<th>$\sigma_{xx}$ peak residual</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.90 $\sigma_{yield}$</td>
<td>0.95 a</td>
</tr>
<tr>
<td>0.1</td>
<td>0.74 $\sigma_{yield}$</td>
<td>0.60 a</td>
</tr>
<tr>
<td>0.2</td>
<td>0.53 $\sigma_{yield}$</td>
<td>0.55 a</td>
</tr>
</tbody>
</table>

The effectiveness of the shot peening treatment depends on the size and depth of compressive residual stresses. Therefore these findings clearly demonstrate how critical the selection of peening parameters such as intensity is. The selection of shot peening intensity should consider the workpiece material post-yield behaviour as this controls not only the magnitude of beneficial residuals but also the extend of the protective layer.

With regard to the problem of multiple elastoplastic indentations two cases are
presented. In the first case the indentations were very close to one another \( (d=2.3a) \), and in the second case they were further apart, \( (d=4.0a) \). Where \( a \) is the radius of the indentation and \( d \) is the centre to centre distance between the indenters. In these cases the assumptions made, methodology and load history, were identical to those used in the single indentation study.

The elastoplastic analysis has provided some interesting results regarding residual stresses. In the centre of the indentations the \( \sigma_{xx} \) residual stress is compressive as expected however, the \( \sigma_{yy} \) residual stress is predominantly tensile on and below the surface, but of a very low magnitude. These observations are shown in Fig.4 & Fig.5, for \( d=2.3a \); Similar results were obtained for \( d=4.0a \). Furthermore, considering the residual stresses at mid-distance between indenters revealed some remarkable trends, especially for large separation between spherical indenters. These residual stress distributions are shown in Fig.6 & Fig.7 for \( d=2.3a \) and Fig.8 & Fig.9 for \( d=4.0a \).

![Graph](image1)

**Figure 4** \( \sigma_{xx} \) Residual Stress Distribution at Centre of Indentation \( d=2.3a, d=4.0a \)

![Graph](image2)

**Figure 5** \( \sigma_{yy} \) Residual Stress Distribution at Centre of Indentation \( d=2.3a, d=4.0a \)
Examination of the $\sigma_{xx}$ residual stress distributions at mid-distance between indenters, Fig.6 & Fig.8, clearly shows that in the case of small separations the stresses are compressive on the surface, attaining a maximum value below the surface. However, for a larger separation the trend changes dramatically and although the surface residual stresses are compressive they rapidly decay to zero. Considering the $\sigma_{yy}$ residual stress components the results are significantly different, Fig.7 & Fig.9. The surface residual $\sigma_{yy}$ stresses are always tensile irrespective of the separation distance. The magnitude of $\sigma_{yy}$ residuals is significantly higher when separation is small, and decreases to near zero as the separation increases.

Evidently, the separation of the indenters affects the residual stresses developed in the target material. The magnitude of residual stresses and the depth of the protective layer are clearly dependent upon the post-yield behaviour of the workpiece material. The higher the strain hardening modulus $H$, the lower the magnitude of the compressive surface and sub-surface residual stresses. With reference to the shot-peening process these results are important as they show that, although incomplete coverage is undesirable, some surface compressive residual stresses could exist even in uncovered areas. However, in practice incomplete coverage has been found to be detrimental, and this could now be explained with the results of elasto-plastic modelling, revealing that in the untreated areas damaging surface tensile residual stresses are present, which can promote crack initiation.

**Figure 6** $\sigma_{xx}$ Residual Stress Distribution at Mid-distance, $d=4.0a$
Figure 7 $\sigma_{yy}$ Residual Stress Distribution at Mid-distance, $d=4.0a$

Figure 8 $\sigma_{xx}$ Residual Stress Distribution at Mid-distance, $d=4.0a$
CONCLUSIONS

The modelling methodology herein presented, utilise advanced Finite Element Methods for the study of shot peening mechanics. It has been demonstrated that the methods developed are capable of dealing with the problem of contact accounting for single and multiple indenters. The elastic solutions of single and multiple indentations have been compared with results from photo-elasticity and the agreement has been found to be very favourable. This has provided confidence in Finite Element modelling.

Elasto-plastic analysis has shown that the strain hardening behaviour of the peened material affects the residual stress field. In fact the higher the strain hardening modulus $H$, the lower the maximum compressive sub-surface residual stress. Another interesting finding is the presence of small magnitude tensile residual stresses in the uncovered area between indentations. It has been revealed that their magnitude dependents upon the distance of separation between indenters. Furthermore, for small separation ratios some compressive sub-surface residual stresses are also present and their magnitude increases as the separation between indentations is reduced. These findings explain the detrimental effects caused by incomplete coverage.

Modelling with Finite Element Methods has been found to be very versatile and a powerful means for investigating the shot-peening process. Although modelling cannot substitute experiments it provides the means for improving understanding of the mechanics of the shot-peening process.
REFERENCES
2. Kyriacou S. and Al-Khaja, J.A. 'Investigation of Shot-Peening and Re-Peening Effects on Partially Fatigued Notched Components', Submitted to ICSP-6