Residual stresses in plastically deformed wires

This study shows that the highest residual stresses occur around the border between the elastic and plastic zones and in the farthest layers from the wire’s axis.

By Fryderyk Knap

Wire and wire-based products comprise the majority in the group of plastically deformed steel products. Often those products work at high external loads. Ropes and cords in lifting equipment, all kinds of springs, cord to reinforce tires, various kinds of strings, wire to reinforce concrete, and other wire products can be included in this category. Behavior of various machine and construction elements and reliability of their operation which guarantee steady and safe work depend mostly on an external layer condition. Residual stress is one of the most substantial parameters that determine the external layer condition.

In cold drawn wires and in wire-based products, as a rule, residual stresses remain after the completion of manufacturing processes. Often, they reach significant values, which in extreme cases even lead to cracking of plastically deformed metal. Residual stresses cause initial effort of a material before an external load is applied to a given product. The resulting stress condition dominating a given product is equal to the sum of external load stresses and residual stresses.

Residual stresses must be considered when determining the real stress, especially when residual stresses have the same sign as the stresses connected with external loads, because stress “summing up” takes place.

Residual stresses in prestressed springs

During cold or hot spring prestressing the external layer of spring would go to a plastic state when tangential stresses reach yield stress value. The central zone, on the other hand, is in an elastic state, and stresses change in the elastic state from a zero value in the wire axis to a maximum value on the border between elastic and plastic zones. If the spring is released from the external load action, then a decrease of external load is accompanied by a linear stress alteration. Overlaying of stresses during unloading on previously existing stress distribution causes residual stresses to remain in relieved material. In external layers the residual stresses have a sense of vector opposite to that of stresses during active loading. In the internal layer residual stresses have the same sign as stresses during active loading. The residual stresses reach the highest values around the border between the elastic and plastic zones during active loading and in a wire near-surface layer.

If we take off an external layer of a thickness $dr$ from the spring, then the twisting moment in the central layer will not be balanced by a twisting moment occurring in a near-surface layer. The twisting moment for the removed external layer, in which the tangential stress $\tau$ operates on a ring of the area $2\pi (r-dr)dr$, is defined by the relationship

$$dM=2\pi \tau (r-dr)^2 dr$$  \hspace{1cm} Eq. (1)

On the other hand the moment $dM$ will cause an elastic strain of the spring when the first layer is removed, and linear alteration of stress-
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es on the spring cross section. If we denote the alteration of stresses on the external surface as \( d\tau \) then

\[
dM = 0.5\pi(r-dr)^3 d\tau
\]

Eq. (2)

Alteration of tangential stresses will cause an alteration of distance between wire coils; when the tangential stress alters by \( d\tau \) the distance between coils will alter by \( db \). Therefore

\[
d\delta = \frac{\pi D^2 d\tau}{2G(r-dr)}
\]

Eq. (3)

If the removed layer thickness \( dr \) is relatively small when compared with a radius \( r \), instead of the \( r-dr \) element the radius value \( r \) may be taken for calculations, and then, based on the relationship of (1), (2) and (3) the result is

\[
\tau = \frac{Gr^2 d\delta}{2\pi D^2 dr}
\]

Eq. (4)

When removing consecutive layers the spring undergoes a deformation which causes an alteration of original state of existing residual stress. To determine the originally existing stress one should note that the dominant stress state in a given moment arises as a result of overlaying stress alterations related to deformations during the removal of consecutive layers on the originally existing stress state. Therefore

\[
2\pi r^2 dr \left( \tau + \frac{2Gr\Delta\delta}{\pi D^2} \right) = \frac{\pi}{2} r^3 d\tau
\]

Eq. (5)

When taking the relationship (3) into regard with Eq. (5), the result is

\[
\tau = \frac{2Gr}{\pi D^2} \left( \frac{rd\delta}{4dr} - \Delta\delta \right)
\]

Eq. (6)

It is apparent from Eq. (6) that one needs knowledge of a modulus of elasticity \( G \), spring diameter \( D \), instantaneous wire diameter \( d \),

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Fig. 2. Internal stresses in springs.

Fig. 3. Test stand scheme.
Spring lead alterations were measured with a toolroom microscope with an accuracy of 0.01 mm. See Fig. 1. The examinations for prestressed springs were carried out in an ambient temperature of 300°C. Examinations have proven that during the removal of consecutive layers the spring lead increases. The residual stresses in particular points on a cross section of wire were determined based on the information derived from Eq. 6. See Fig. 2. Examinations and calculations have revealed that in a near-surface layer the highest stresses occur on a wire’s surface. As one moves away from the wire’s surface the residual stresses decrease and at some distances from the wire’s surface they drop to zero, and then change the sign, and their absolute value increases as one moves further away from the wire’s surface. The removal of consecutive layers was carried out until the radius of about 0.45 mm was reached. Within the examined range the layer removal was satisfactorily uniform.

To effectively determine the influence of residual stresses that remain in the spring after prestressing, spring fatigue strength examinations were carried out on the test stand based on a vehicle engine timing gear system. See Fig. 3. Examined springs were placed in head seats, and initial loading was achieved through compressing the spring with screw and nuts by a specified value. Basic loading was achieved with projections of the distribution shaft that displace intermediate elements and the screw with nuts by a desired value. The distribution shaft was rotated by an electric motor through a belt transmission. To achieve various values of basic loading the distribution shaft projections were ground to a set value. The stand made it possible to effectively reflect the actual conditions of spring loading during their operation. The springs assigned to be pre-

relation of spring lead differential $d\delta$ to wire diameter differential $dr$, and spring lead alterations in previous stages of consecutive layer removal to determine tangential stresses $\tau$.

**Experimental determination of internal stresses in prestressed springs**

Test springs were made from 3 mm diameter drawn wire of D65 steel containing 0.67 percent carbon; the spring diameter $D$ was 23.3 mm and spring lead was 13.32 mm. Consecutive layers of spring were removed by hydrochloric acid etching, and their thicknesses were determined based on spring weight alterations.

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**Fig. 4. Comparison of number of cycles until conventional and prestessed springs are destroyed.**

**Fig. 5. Residual stresses in curve formed elements.**

**Fig. 6. Residual stresses experimentally determined in curve element.**
stressed the load of 13.32 mm, and the overstrained ones—9.43 mm; the load of unprestressed springs was 9.45 mm. The pitch diameter was 23.3 mm, and the number of coils was two. The conventional yield limit determined in wire tensile test was 1380 MPa, and the tensile strength was 1580 MPa. Unprestressed springs and springs assigned to be prestressed were stress relieved annealed at 350°C for 30 minutes. Fatigue tests were carried out at various levels of preliminary loading (deflection) and basic loading. In the first variant the preliminary deflection was 2 mm, and the basic one, 7 mm; in the second variant—1 mm and 7 mm correspondingly, while in the third variant—1 mm and 6 mm. Deflection by 1 mm corresponded to tangential stresses equal to 70.4 MPa. Total effort factor related to the plasticity limit was 0.79 in the first variant, and 0.62 in the third one. The general concept was to carry out examinations at a relatively high stress level. In a theoretical analysis it was proved that the highest beneficial influence of prestressing on resulting stress will occur at a relatively low level of stresses derived from external loading. Instead, at higher stress levels the prestressing influence is less effective. Thus, if someone proves that there is a beneficial influence at high external stresses then it may be expected that a more beneficial one will occur at lower stresses.

The number of cycles needed to destroy prestressed and unprestressed springs is listed in Fig. 4 where it can be seen that the fatigue strength of prestressed springs was considerably higher than that of conventional springs. Effectiveness of influence depends, to a high degree, on the level of stresses resulting from external loading. Higher effectiveness of prestressing influence on fatigue strength was achieved for lower level of stresses. In all cases spring fatigue fractures began from the external surface of the internal side of the spring. On that surface the stresses are higher than on the external side of the spring which causes its faster fatigue.

Examinations have shown, according to expectations, that prestressing increases the spring material’s resistance to fatigue, and so spring prestressing allows permissible stresses to increase. Overall spring dimensions and volume may be diminished.

**Residual stresses in curve formed elements**

During forming of many wire-based products (springs and products with fragments in the form of an arc with a relation of arc curvature radius to wire radius of the order of 10 and less) the plastic layers dominate, while the range of elastic zone is relatively small. In plastic stretched and compressed zone the circumferential stresses reach yield stress values, while in the elastic zone they alter linearly from a zero value in the wire axis to a value equal to a yield limit on a line that separates the elastic zone from the plastic zone. During relieving, previously stretched fibers experience compression, while compressed layers experience stretching. Deformations during relieving have an elastic character and change linearly depending on a given point distance from the neutral axis. Overlaying of relieving stresses on previously existing stress states during active loading does not lead to a complete relieving and some residual stresses remain in the deformed material. See Fig. 5. In the wire layers most distant from the neutral axis residual stresses remain with a sign consistent with the sign of stresses that occur during relieving. While in remaining zones the residual stress sign is consistent with the one of stresses that occur during active loading, and on the border between elastic and plastic zones the residual stresses reach extreme values, close to the yield limit.

Based on moment and force equilibrium conditions and physics relations between stresses and deformations one obtains the following form of equations to determine internal stresses:

Original value of stresses dominating in a material prior to removing layers will be obtained from the equation:

\[\sigma_{i(a)} = - \frac{EI_{(a)}}{(r_0 - y_{(a)} - a)(R_0 + y_{(a)})^2} b(a)\]

\[\frac{d\delta}{da} + E \int_0^{\xi} \frac{r_0 - y_{(a)} - a}{(R_0 + y_{(a)})^2} d\xi + \]

\[+ E \int_0^{\xi} \frac{I(\xi)}{(r_0 + y_{(a)} - \xi)(R_0 + y_{(a)})^2} \frac{d\delta}{d\xi} d\xi\]

Eq. (7)

And in the other half of the wire the stresses are determined based on the equation

\[\sigma_{i(a)} = - \frac{EI_{(a)}}{(r_0 - y_{(a)} - a)(R_0 + y_{(a)})^2} b(a)\]

\[\frac{d\delta}{da} + E \int_0^{\xi} \frac{r_0 - y_{(a)} - a}{(R_0 + y_{(a)})^2} d\xi + \]

\[+ E \int_0^{\xi} \frac{I(\xi)}{(r_0 + y_{(a)} - \xi)(R_0 + y_{(a)})^2} \frac{d\delta}{d\xi} d\xi\]

Eq. (8)

The moment of inertia of a wire’s cross section in relation to the axis as a function of a given surface position may be calculated from the equation

\[I(\xi) = 2 \left[ \frac{\gamma_{(a)}}{2} \sqrt{\xi}(2r_0 - \xi) + r_0^2 \arcsin \frac{r_0 - \xi}{r_0} \right] + \]

\[- \frac{2}{3} \gamma_{(a)} \sqrt{\xi}(2r_0 - \xi) + \frac{r_0^2}{4} \sqrt{\xi}(2r_0 - \xi)^2 + \]

\[+ \frac{r_0^2}{8} \left( \frac{r_0 - \xi}{\sqrt{\xi}(2r_0 - \xi)} + r_0^2 \arcsin \frac{r_0 - \xi}{r_0} + \frac{r_0^2}{16} \right)\]

Eq. (9)

The cross section center of gravity undergoes displacement during the removal of consecutive layers. The position of the center of gravity was determined through an equation of static moments in relation to neutral axis.
The experimental examinations of residual stresses were carried out on wire samples made of D65 steel with a 2.6 mm diameter and coiled on an arbor of 20 mm diameter. Consecutive layers were removed from the wire by grinding, beginning with the external surface of the upper half of the wire, while grinding on the lower half of the wire was begun from an internal surface. When a consecutive layer of a thickness of about 0.1 mm had been removed, the wire diameter was measured on a side opposite in relation to removed layers.

Measurements and calculations showed that in a wire coiled in a disc there are residual stresses non-uniformly distributed on the wire's cross section. The highest residual stresses exist in the zone around the border between elastic and plastic zones during active forming, and in layers the farthest from the wire axis. See Fig. 6.

**Conclusions**

1. Residual stresses—with a sign opposite to stresses occurred during spring forming—remain in the farthest layers from the wire's axis in overstrained springs.
2. The prestressed springs show higher fatigue strength than unprestressed springs.
3. Residual stresses remain after non-uniformly relieving stress distributed on a wire's cross section in curve formed wires. The highest residual stresses occur around the border between elastic and plastic zones during wire active loading, and in the farthest layers from the wire's axis.

**References**