"The Modeling of shot movement in portable and pneumodinamic equipments."

Michail Matlin, Valery Mosseiko, Viatcheslav Mosseiko

Volgograd State Technical University

At shot-blast or grit blasting treatment for surface-hardening of detail, the angle of shot incidence counted from normal to surface, in most cases differs from $0^\circ$ by virtue of the complex shaped form of many details, and also conical or fan form of the flow of falling shot and its reciprocal percussion.

In this connection the research of kinematics parameters of shot after its rebound, necessary at the designing technical means of treatment by the shot [1], and the definition of parameters of elasto-plastic contact deformation and surface hardening at angles of shot incidence different from $0^\circ$ (slanting impact) represent a certain interest.

The experimental researches of the angle of reflection $\beta$ single pellet from a processable surface at various angels of its progressive falling $\alpha$ have shown, that these angles essentially (in two and more times) differ from each other, coinciding only at $\alpha=\beta=0^\circ$ and $\alpha=\beta=90^\circ$. The greatest difference of angles of reflection and incidence is observed in the diapason of angles of incidence from $-10^\circ$ up to $-60^\circ$ with maximum at $-30^\circ$ (fig.1).

The analysis of the surface formed after the impact of casting pipe has shown that at small and close to small angles of incidence, at the bottom of indentation a surface relief of pellet is imprinted, testifying its rolling at the impact. At great angels of incidence all the surface of indentation is covered with banded traces of pellet at its bottom. At some intermediate angles $\alpha$ at the bottom of the indentation, simultaneously both the traces of sliding and rolling, located consistently on the course of the pellet are found (fig.2).

With the purpose of theoretical definition of kinematics parameters of the shot, we shall consider a slanting unelastic impact by the round pellet, examined as a firm sphere of mass $m$, on a rough barrier. For definiteness, we consider its movement before the impact as flat with an initial speed of the center of mass $U_0$ under the angle $\alpha$ from normal to the barrier and angular speed
At the moment of the impact on the part of the barrier in the point $K$ of contact, such an instant percussive force $F$ is put (reaction of the barrier), that the movement of sphere after the impact still remains flat in the result (fig.3).

\[ \beta, \text{grad} \]

![Graph showing dependence of angles of reflection $\beta$ from angels of incidence $\alpha$ of copper-plated pellet by the diameter of 4,5 mm (initial speed 105 m/s): 1 – processed surface – steel HRB100; 2 – steel HRB71; 3 – silumin; 4 – lead.]

Fig.1. Dependence of angles of reflection $\beta$ from angels of incidence $\alpha$ of copper-plated pellet by the diameter of 4,5 mm (initial speed 105 m/s): 1 – processed surface – steel HRB100; 2 – steel HRB71; 3 – silumin; 4 – lead.

![Imprints from the impact with steel shot by the diameter 4,5 mm on the surface of lead at various angles of incidence: 1 – angles of incidence $\alpha=15^\circ$; 2 – $\alpha=45^\circ$; 3 – $\alpha=75^\circ$.]

Fig.2. Imprints from the impact with steel shot by the diameter 4,5 mm on the surface of lead at various angles of incidence: 1 – angles of incidence $\alpha=15^\circ$; 2 – $\alpha=45^\circ$; 3 – $\alpha=75^\circ$.

Then as the initial equations for definition of the speed $U$ of the center of sphere weights and its angular speed $\omega$ after the impact, we use the equations of mechanics, expressing the theorem of pulse and theorem about the changing of the moment of quantity's movement in projections on the axis of coordinates (the mass of the barrier is considered incomparably more than the mass of the sphere, wave phenomenon are
neglected, the impact is considered as the spasmodic process of infinitely little duration) [2]:

\[ m(U_x - U_{ox}) = S_x, \quad (1) \]
\[ m(U_y - U_{oy}) = S_y, \quad (2) \]
\[ J_z^c (\omega - \omega_0) = rS_x, \quad (3) \]

where: \( S_x \) and \( S_y \) – are the projection of force impulse \( F \) on the axis of coordinates;
\( U_{ox} \) and \( U_{oy} \) and \( U_x \) and \( U_y \) - are the projection of the speed of the center of sphere;
\( J_z^c \) - is the moment of the sphere inertia concerning its central axis.

Further, we shall take into account, that the projections of the speed \( U \) of the center of sphere weight and the speed \( V \) of its point \( K \), according to the theorem of speed body points, making flat movement, can be expressed as follows:

\[ U_{ox} = V_{ox} - \omega_0 r, \]
\[ U_x = V_x - \omega r, \]
\[ U_{oy} = V_{oy}, \quad U_y = V_y. \quad (4) \]
Here normal making $V_c$ and $V_{oy}$ speeds of point $K$ of sphere within the frames of general theory of unelastic impact from classical mechanics [2] will correspond as follows:

$$V_y / |V_{oy}| = -k \text{ или } V_y = -kV_{oy}.$$  \hspace{1cm} (5)

where $k$ is the factor of restoration at the impact.

The additional equation, making the system (1-5) closed, can be received from the analysis of force interaction of the sphere with the barrier in the point $K$. We shall consider three possible cases, following from this analysis. The first case without practical meaning will correspond to an absolute smooth surface of the barrier; thus decisions, interesting for us, will be like:

$$S_x = 0, S_y = -m(1 + k)U_{oy}, U_x = U_{ox}, U_y = -kU_{oy}, \omega = \omega_0.$$ \hspace{1cm} (6)

and the required angle of reflection $\beta$ can be determined through its tangent

$$\tg \beta = \frac{U_x}{U_y} = \frac{tga}{k}.$$ \hspace{1cm} (7)

The second, practically important case correspond to an absolute rough barrier, interfering sliding of the sphere at the impact; the appropriate decision will be like:

$$S_x = -\frac{2}{7} m(U_{ox} + \omega_0r), S_y = -m(1 + k)U_{oy},$$

$$U_x = \frac{5}{7} U_{ox} - \frac{2}{7} \omega_0r, U_y = -kU_{oy}, \omega = \frac{2}{7} \omega_0 - \frac{5}{7} \frac{U_{ox}}{r}.$$ \hspace{1cm} (8)

Thus, the angle of reflection $\beta$ for progressively falling sphere ($\omega_0=0$) can be determined as:

$$\tg \beta = \frac{5}{7} \frac{tga}{k}.$$ \hspace{1cm} (9)

The second case is possible at small meaning of horizontal components of the sphere speed point $K$ (for example, at small angles of its incidence and small angular speeds), when arising in the point of contact striking power of friction does not surpass its limited meaning, higher of which the slipping motion comes. In this connection is necessary to mean, that the field of remissible angles $\alpha$ is in dependence (9) – only their small and
close to them meanings.

The third, also practically important case, will correspond to the slipping motion in the point \( K \) for all the time of the impact if there is a final friction. If we consider in the first approximation, that the projection \( S \) of the pulse of striking power is proportional to the projection of pulse \( S_y \) or \( S_x = \pm fS_y \), where \( f \) is the factor of friction at the impact, then the appropriate decision will be like:

\[
S_x = -mf(1+k)|U_{oy}|, \quad S_y = -m(1+k)\hat{U}_{oy}, \quad U_x = \hat{U}_{ox} - f(1+k)\hat{U}_{oy},
\]

\[
U_y = -k\hat{U}_{oy}, \quad \omega = \omega_0 - \frac{5f(1+k)}{2r} |\hat{U}_{oy}|
\]

The angle of reflection \( \beta \) for the progressive falling sphere can be determined as:

\[
tg\beta = \frac{tg\alpha - f(1+k)}{k}.
\]

The field of permissible meaning of angles \( \alpha \) in the formula (11) is the angles close to \( 90^\circ \), when at the impact of the progressive falling sphere, its pure sliding in the point of contact is observed (moreover the negative meanings of angles \( \beta \) at small \( \alpha \) and \( f>0 \), contradiction to physical sense are excluded). Thus for the analysis of the sphere rebound phenomenon in the whole diapason of angels \( \alpha = 0...90^\circ \) it is necessary to use the dependence (9) at small angles; the dependence (11) at angles close to \( 90^\circ \).

For intermediate angles \( \alpha \), when at the impact of the sphere its slipping motion in the point of the contact, and then the rolling around the bottom of the formed indentation is observed, it is necessary to build intermediate curves, smoothly connecting dependences on (9) and (11), as for instance, is shown by dashed lines in (fig.4) for factor of friction \( f=0.2 \).

As it is visible from the comparison of experimental data in (fig.1) and theoretical curves in (fig.4), they give significant qualitative coincidence. It gives the reason to consider the offered model of the oblique unelastic impact of the firm sphere about a rough barrier satisfactory.

Thus, indirect determination of orientation meaning of factor of friction and restoration is obviously possible at the impact by direct comparision of experimental and theoretical curve of angels of reflection without some special experiments.
Fig. 4 The dependence of angle of reflection $\beta$ to angle of incidence $\alpha$ for various factor of restoration $k$ and the factor of friction $f=0,2$:
1. $k=0,1$; 2. $k=0,2$; 3. $k=0,4$; 4. $k=0,6$; 5. $k=0,8$; 6. $k=1,0$

Bibliographical list
