Probabilistic Assessment of Shot Peening Impact Coverage

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Introduction
In shot peening, compressive residual stresses are induced through impact events between media and the surface of a component. When designing a shot peening machine and specifying operational parameters, practitioners often aim to achieve full and even coverage through sufficiently long cycle times and careful positioning of the peening nozzle with respect to the treated surface. Mass flow rate, peening time, and blast pressure are particularly important when considering impact coverage on a component. Over a peening cycle, a discrete number of particles leave the nozzle, each with an associated mass. For a given substrate material, particle mass, and media hardness, the size of the surface dimple left by an impact is determined by the particle's velocity, meaning the distribution of dimple coverage is directly related to the uniformity of particle mass flux.

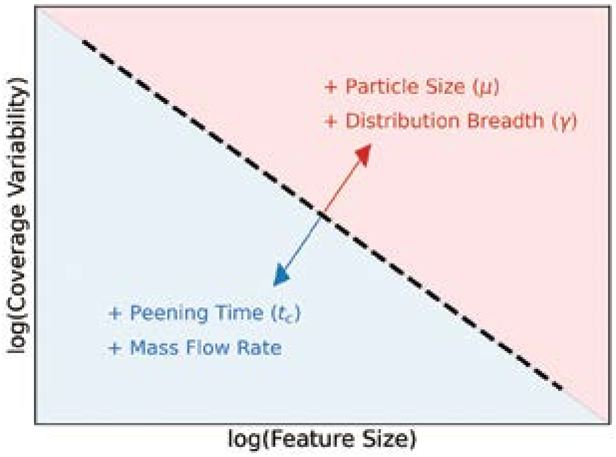

Figure 1. Peening time, mass flow rate, and media size distributions contribute to coverage variability across scales of scrutiny.

Considering shot media as point masses with randomly chosen impact locations upon a component of a fixed area, denoted A_{part}, the average mass flux ($\dot{m}$) is equal to the summation of all particle mass contributions over a cycle, divided by the peening time, t_c, and A_{part}, or equivalently mass flow rate divided by A_{part}. Within control limits, $\dot{m}$ can be considered a constant derived directly from mass flow rate. On the other hand, let M_A be the total mass of media that impacts within an arbitrary region with area A, shown in Equation 1.

Equation 1.

$$M_A = \sum_{i=0}^{n_A} m_p^i$$

M_A depends on two distributed quantities, the number of particles that impinge the region (n_A), and the mass of each particle (m_p^i). The expected, or average, value of a random quantity (denoted E[·]), is the probability-weighted summation of all possible values the variable can take. Wald's identity (Wald, 1944), (Ross, 1996) enables a simplified calculation of $E[M_A]$, separating the contributions of the independent variables n_A and m_p^i. Thus, $E[M_A]$ can be expressed as:

Equation 2.

$$E[M_A] = E[n_A] \cdot E[m_p^i]$$

Variance (denoted *Var*[·]) is a measure of the breadth of a distribution. Like Wald's identity, the Blackwell-Girshick equation (Blackwell & Girshick, 1979) allows for the separation of contributions from n_A and m_p^i with respect to variance. $Var[M_A]$ can then be expressed as:

Equation 3.

$$Var[M_A] = E[n_A] \cdot Var[m_p^i] + Var[n_A] \cdot E[m_p^i]^2$$

Relative standard deviation (denoted *RSD*[·]) is the ratio of standard deviation, or the square root of variance, to average, a dimensionless quantity that expresses the proportional variability of a measure with respect to its mean value. In this context, $RSD[M_A]$ is certainly correlated to the variability in total work imparted onto corresponding features across runs of components. The goal of this report is to apply probabilistic reasoning to characterize a spatial distribution in impact coverage based on cumulative mass over the surface of a component. Specifically, deriving expressions for $E[M_A]$, $Var[M_A]$, and $RSD[M_A]$ to provide perspective on how operational parameters relate to surface treatment uniformity across scales of scrutiny.

Spatial Uniformity of Mass Flux
In a previous Shot Peener report entitled "Characterization of Particle Size and Shape Distributions for Shot Peening Media" (Feltner, Gruninger, Canty, & Mort, 2024), we explored volume weighted distributions in peening media size and shape measured using dynamic image analysis (DIA). This work demonstrated the suitability of a lognormal distribution for describing size in relation to mass sieving. In the current work, DIA is used to

calculate number weighted distributions for probability-based impact coverage modeling.

A lognormal distribution in area equivalent radius (R) implies that $\ln(R)$ is normally distributed with a dimensionless geometric mean (μ) and standard deviation (γ). To be consistent with the notation used in our previous work, $\mu=ln(d_{gN}/2)$, the natural log of half the number weighted geometric mean particle diameter, while $\gamma=ln(\sigma_{gN})$, the natural log of the number weighted geometric standard deviation. The expected value of a particle's radius is $exp(\mu+\gamma^2/2)$. The mass of a particle scales according to its radius cubed, meaning mass is lognormally distributed with mean 3μ and standard deviation 3γ. Assuming a constant density ρ, the expected mass of a particle is:

Equation 4.
$$E[m_p^i] = \frac{4\pi\rho \cdot exp(3\mu + \frac{9\gamma^2}{2})}{3}$$

Shown in Equation 5, the average number flux of impacts ($\dot{n}$) is the mass flux divided by the expected mass per particle. Similar to $\dot{m}$, $\dot{n}$ can be considered constant within control limits. $\dot{n}$ is inversely proportional to the mean radius cubed, with a conspicuous additional inverse dependence on γ.

Equation 5.
$$\dot{n} = \frac{3 \cdot \dot{m}}{4\pi\rho \cdot exp(3\mu + \frac{9\gamma^2}{2})}$$

To illustrate the consequences of this dependence to the relative uncertainty in impact coverage, consider the case where $n \approx \dot{n}\ t_c \cdot A_{part}$ particles are assigned random impact locations within an area of size A_{part}. For a measurement area A that is a subsection of A_{part}, the probability that any particle is contained within is $p_0 = A/A_{part}$, and the probability a particle is excluded is $q=1-p_0$. The probability that the measured number of particles within A (n_A) is exactly equal to k is equal to the number of ways to choose k particles from n, multiplied by p_0^k (the probability that all k particles are within A) and q^{n-k}(the probability that all other particles are not within A), shown in Equation 6.

Equation 6.
$$P(n_A = k) = \frac{n!}{k!\,(n-k)!} p_0^k q^{n-k}$$

This is known as the binomial distribution, a fundamental construct in probability theory used to describe the number of successes and failures in experiments with independent and identically distributed trials. When the total number of impacting particles is large, the binomial distribution converges to a Poisson distribution (Ross, 1996):

Equation 7.
$$P(n_A = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Where $\lambda = \dot{n}\ At_c$. The Poisson distribution has an expected value and variance of $E[n_A] = Var[n_A]=\lambda$, and is commonly used to model counting processes. Applied to peening, this mathematical formulation provides an exact probability distribution for the number of particles contained within a region based on operational parameters and media size.

Leveraging the equality of expected value and variance of a Poisson distribution, Equation 3 becomes:

$$Var[M_A] = E[n_A] \cdot (Var[m_p^i] + E[m_p^i]^2)$$

The definition of variance states that:

$$Var[M_A] = E[n_A] \cdot (Var[m_p^i] + E[m_p^i]^2)$$

(Ross, 1996), leading to a simplified expression for $Var[M_A]$:

Equation 8.
$$Var[M_A] = E[n_A] \cdot E[{m_p^i}^2]$$

Applying the expected value of the Poisson distribution for N_A and the second moment $E[{m_p^i}^2]$

of the lognormal size distribution leads to:

Equation 9.
$$Var[M_A] = (\dot{n}At_c) \cdot \left(\left(\frac{4\pi\rho}{3}\right)^2 \cdot exp(6\mu + 18\gamma^2)\right)$$

Substituting Equation 5 yields an expression for $Var[M_A]$ in terms of $\dot{m}$:

Equation 10.
$$Var[M_A] = \left(\frac{4\pi\rho\dot{m}At_c \cdot exp(6\mu + 18\gamma^2)}{3 \cdot exp(3\mu + \frac{9\gamma^2}{2})}\right)$$

Simplifying to a final expression for $Var[M_A]$:

Equation 11.
$$Var[M_A] = \left(\frac{4}{3}\pi\rho\dot{m}At_c \cdot exp(3\mu + \frac{27}{2}\gamma^2)\right)$$

$E[M_A]$ is simply equal to $\dot{m}At_c$, leading to a closed form expression for relative standard deviation:

Equation 12.
$$RSD[M_A] = 2\sqrt{\frac{\pi\rho \cdot exp(3\mu + \frac{27}{2}\gamma^2)}{3\dot{m}At_c}}$$

To validate the Poisson distributions application to predicting peening mass flux uniformity, consider three media size distributions for conditioned cut wire 32; 1) idealized monodisperse ($\mu= ln(463), \gamma = ln(1)$), 2) as-manufactured ($\mu$- $ln(463)$, γ - $ln(1.071)$), and 3) working mix (μ- $ln(359.2)$, γ - $ln(1.422)$). As-manufactured and working mix media samples were obtained and measured with DIA as part of a previous Purdue University School of Materials Engineering senior capstone project (Kelly, Keuneke, McLaughlin, & Schroader, 2021). Linearized lognormal fits for both are summarized in Figure 2 (page 20). Overall, most working mix particles are smaller than the as-manufactured, though the working mix has a significantly broader distribution.

Using those particle size distributions, a relatively simple Monte Carlo procedure can be performed to simulate impact coverage uniformity numerically. Assuming a constant mass flow rate of 20 kg/min, A_{part} is equal to 0.03 m^2, and a t_c of either 10 or 50 s, each particle size distribution is repeatedly sampled on a number basis until the cumulative mass of media is greater than or equal to the product of mass flow rate and peening time. The sampled particles are then assigned random (x,y)

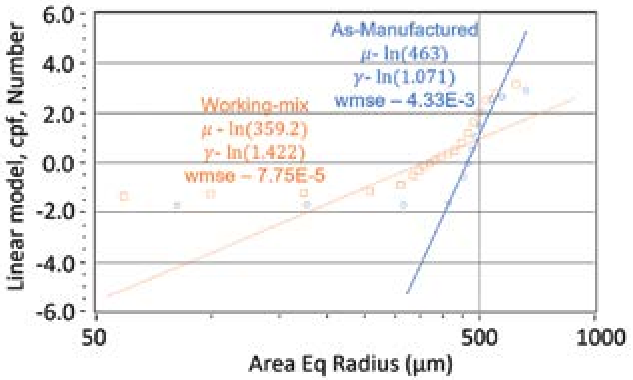

Figure 2. Linearized lognormal fits for area equivalent radius of as-manufactured and working mix CCW32 media.

impact locations within a region of size A_{part}. A_{part} is recursively subdivided into grids of progressively smaller measurement regions, enabling direct calculation of the distribution of M_A at each measurement size. The standard deviation of the set of M_A values divided by its mean yields $RSD[M_A]$ as a function of measurement area. The use of a number-weighted distribution in area equivalent radius is critical to this exercise, ensuring that all particles have the same likelihood of impinging a component, regardless of size.

Shown in Figure 3 is a comparison between Poisson process predictions for $RSD[M_A]$ with Monte Carlo results, demonstrating a clear agreement between the two. In capturing variability in impact coverage, the results of this study suggest that monodisperse is an appropriate approximation for as-manufactured media, while the breadth of the working-mix distribution contributed to a lower number-flux of impacts on average and hence, greater variability in impact coverage across scales of scrutiny.

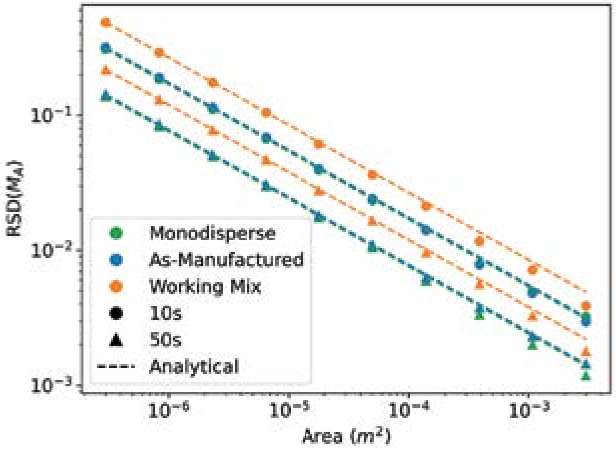

Figure 3. Comparison of analytical prediction with Monte Carlo results for relative variability in cumulative mass as a function of area.

In standard peening processes, with controlled centerlines of air pressure, feed rate, and working mix media size, this model provides insight to the effective impact distribution at relevant scales ranging from slightly larger than the media size up to full parts. It is important to note the limitations of this Poisson process-based model with respect to operational parameters. The Poisson approximation of the binomial distribution relies on a large total number of impacts. Though divergence from the analytical prediction was not observed in this study, processes with especially low average mass fluxes coupled with broad particle size distributions could violate the assumption of independence between the number of impacting particles and the mass of each particle. Additionally, this model assumes that particles are point masses; quantifying a spatial distribution of mass at sub-dimple length scales can be ambiguous.

Conclusions

The Poisson model can provide a starting point for predicting variability in residual stress fields across treated surfaces. Critical features of many peened components, for example axle gear roots and turbine leading edges, fall between the component size and a dimple diameter, the ideal range for Poisson model validity. The Poisson model describes coverage as a counting process; hence it is important to obtain number-based media size distributions, for example using DIA. Results suggest that peening time, mass flow rate, and media size can be used to control uniformity of coverage. We seek to use this model to aid in the design of peening processes that achieve desired stress profiles minimizing variability in critical regions of a component. More broadly, we see this work as a starting point toward the development of advanced statistical tools for linking operational parameters and transient particle size and shape distributions to spatial and temporal uniformity in both surface topography and residual stress fields.

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